No. of Printed Pages : **2**

Sl. No.

B3.2-R4 : DISCRETE STRUCTURES

NOTE :

1. **Answer question 1 and any FOUR from questions 2 to 7.**
2. **Parts of the same question should be answered together and in the same sequence.**

Time : 3 Hours **Total Marks : 100**

1. (a) What is the covering relation of the partial ordering $\{(a, b) \mid a \text{ divides } b\}$ on $\{1, 2, 3, 4, 6, 12\}$?

(b) Let f and g be the functions from the set of integers to the set of integers defined by $f(x) = 4x + 7$ and $g(x) = 7x + 4$. What is the composition of f and g ? What is the composition of g and f ?

(c) Let G be the grammar with vocabulary $V = \{S, A, a, b\}$, set of terminals $T = \{a, b\}$, starting symbol S, and productions $P = \{S \to aA, S \to b, A \to aa\}$. What is $L(G)$, the language of this grammar ?

(d) The bit strings for the sets {1, 2, 3, 4, 5} and {1, 3, 5, 7, 9} are 11 1110 0000 and 10 1010 1010, respectively. Use bit strings to find the union and intersection of these sets.

(e) Suppose that a connected planar simple graph has 20 vertices, each of degree 3. Into how many regions does a representation of this planar graph split the plane ?

(f) Suppose that there are eight runners in a race. The winner receives a gold medal, the second place finisher receives a silver medal and the third place finisher receives a bronze medal. How many different ways are there to award these medals, if all possible outcomes of the race can occur and there are no ties ?

(g) What are the equivalence classes of 0 and 1 for congruence modulo 4 ?

(7x4)

2. (a) Let f be the function from the set of all non-negative real numbers to the set of all non-negative real numbers with the function $f(x) = x^2$. Is f invertible ?

(b) Prove by contradiction that there is no greatest even integer.

(c) Use generating functions to determine the number of different ways 10 identical balloons can be given to four children if each child receives at least two balloons.

(6+6+6)

3. (a) What do the statements $\forall x < 0(x^2 > 0)$, $\forall y \neq 0(y^3 \neq 0)$ and $\exists z > 0(z^2 = 2)$ mean, where the domain in each case consists of the real numbers ?

(b) Let S be a set of n integers. Show that there is a subset of S, the sum of whose elements is a multiple of n.

(c) Use truth table, find the values of the Boolean function represented by

$$f(x, y, z) = x(yz + \overline{yz}).$$

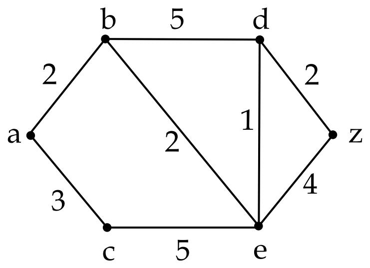
(6+6+6)

4. (a) Show that whenever $n \geq 3$, $f_n > \alpha^{n-2}$ where $\alpha = \frac{(1+\sqrt{5})}{2}$.

(b) Show that $(p \wedge q) \rightarrow r$ and $(p \rightarrow r) \wedge (q \rightarrow r)$ are not logically equivalent.

(c) Prove that the Big-Oh of $T(n) = n^3 + 20n + 1$ is $O(n^3)$.

5. (a) Find the length and shortest path between a and z in the following weighted Graph. **(6+6+6)**



(b) Data are transmitted over a particular Ethernet network in blocks of 1500 octets (blocks of 8 bits). How many blocks are required to transmit the following amounts of data over this Ethernet network ? (Note that a byte is a synonym for an octet, a kilobyte is 1000 bytes and a megabyte is 1,000,000 bytes).

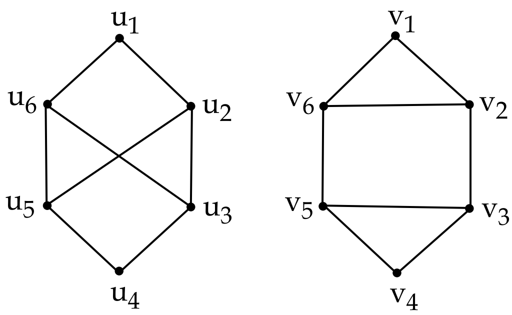
(i) 150 kilobytes of data
(ii) 384 kilobytes of data
(iii) 1.544 megabytes of data
(iv) 45.3 megabytes of data

6. (a) Find the generating function, also called enumerator, for permutation of n objects with the following specified conditions. **(9+9)**

(i) Each object occurs at the most twice.
(ii) Each object occurs at least twice.
(iii) Each object occurs at least once and the most k times.

(b) Construct a Turing machine that computes the function $f(n) = n + 2$ for all non-negative integers n.

7. (a) Determine whether the following graph shown in figure are isomorphic. **(10+8)**



(b) Find all solutions of the recurrence relation $a_n = 6a_{n-1} - 9a_{n-2}$, $n \geq 2$. What is the solution with $a_0 = 1$, $a_1 = 4$? **(10+8)**

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