

BE10-R4 : APPLIED OPERATIONS RESEARCH**NOTE :**

1. Answer question 1 and any FOUR from questions 2 to 7.
2. Parts of the same question should be answered together and in the same sequence.

Time : 3 Hours

Total Marks : 100

1. (a) A company must produce at least 200 units of a product consisting of ingredients A and B. Ingredient A costs Rs. 3 per unit and ingredient B costs Rs. 8 per unit. Not more than 80 units of A can be used and at least 60 units of B must be used. Formulate an LPP to determine the number of units of A and B to be used in order to minimize cost.

- (b) For what value of λ , the game with the following matrix is strictly determinable ?

		Player B		
		B ₁	B ₂	B ₃
	A ₁	λ	6	2
Player A	A ₂	-1	λ	-7
	A ₃	-2	4	λ

- (c) The transportation problem in the following table gives a degenerate basic solution. Suppose that the multipliers associated with this solution are $u_1=1$, $u_2=-1$, $v_1=2$, $v_2=2$ and $v_3=5$ and that the unit cost for all (basic and nonbasic) zero x_{ij} variables is given by $C_{ij} = i + j\theta$, $-\infty < \theta < \infty$.

				Supply
	10	10	10	10
	20	20	20	40
Demand	10	20	20	

If the given solution is optimal, determine the associated optimal value of the objective function.

- (d) Write the dual of the following Primal problem :

$$\text{Max. } z = x_1 + x_2$$

$$\text{s.t. } 2x_1 + x_2 = 5; 3x_1 - x_2 = 6; x_1, x_2 \geq 0$$

- (e) An oil engine manufacturer purchases lubricants at the rate of Rs. 42 per piece from a vendor. The requirement of these lubricants is 1800 per year. What should be the order quantity per order, if the cost per placement of an order is Rs. 16 and inventory carrying charge per rupee per year is only 20 paisa ?

- (f) Customers arrive at a sales counter managed by a single person according to a poisson process with a mean rate of 20 per hour. The time required to serve a customer has an exponential distribution with a mean of 100 seconds. Find the average waiting time of a customer in the queue.
- (g) Construct a network for each of the projects whose activities and their precedence relationships are given below :

Activity	A	B	C	D	E	F	G	H	I	J	K
Predecessor	-	-	-	A	B	B	C	D	E	H, I	F, G

(7x4)

2. (a) A company plans to fill four positions and it decides to conduct aptitude tests and interviews for the same. While the aptitude tests are conducted by people from the clerical positions, the job interviews are held by the personnel from the management cadre. The job interviews immediately follow the aptitude test. The time required (in minutes) by each of the positions is given here,

Position	P1	P2	P3	P4
Aptitude Test	100	110	140	120
Job interview	70	90	80	110

If it is desired to minimise the waiting time of the management personnel, in what order the position filling be handled ?

- (b) Apply the method of Steepest Decent to minimize the function $f(x, y) = 4x^2 - 4xy + 2y^2$ with initial point $X_0 = (2, 3)$. Perform only two iterations.

(9+9)

3. (a) An enterprise has three factories at locations A, B and C which supplies three warehouses located at D, E and F. Monthly factory capacities are 10, 80 and 15 units respectively. Monthly warehouse requirements are 75, 20 and 50 units respectively. Unit shipping costs (in rs.) are given below :

		Warehouses		
		D	E	F
Factory	A	5	1	7
	B	6	4	6
	C	3	2	5

The penalty costs for not satisfying demand at the warehouse D, E and F are Rs. 5, Rs. 3 and Rs. 2 per unit respectively. Determine the initial basic feasible solution using VAM and also test the solution for optimality.

- (b) Solve the following assignment problem to find the maximum total expected sale.

		Area			
		I	II	III	IV
Salesman	A	42	35	28	21
	B	30	25	20	15
	C	30	25	20	15
	D	24	20	16	12

(9+9)

4. (a) Solve the following LPP using Big-M method :

$$\text{Max. } z = 3x_1 + 2x_2 + 3x_3$$

$$\text{s.t. } 2x_1 + x_2 + x_3 \leq 2; \quad 3x_1 + 4x_2 + 2x_3 \geq 8 \quad x_1, x_2, x_3 \geq 0$$

- (b) The following matrix represents the payoff to P_1 in a rectangular game between two persons P_1 and P_2 :

$$P_1 \begin{matrix} & P_2 \\ \begin{bmatrix} 8 & 15 & -4 & -2 \\ 19 & 15 & 17 & 16 \\ 0 & 20 & 15 & 5 \end{bmatrix} \end{matrix}$$

By the notion of dominance, reduce the game to 2×4 game and solve it graphically.

(8+10)

5. (a) Develop the Branch and bound tree for the following problems. Use x_1 as the branching variable at node 0.

$$\text{Max. } z = 5x_1 + 4x_2$$

$$\text{s.t. } 3x_1 + 2x_2 \geq 5; \quad 2x_1 + 3x_2 \geq 7; \quad x_1, x_2 \geq 0 \text{ and are integers.}$$

- (b) Cars arrive at a petrol pump with exponential inter-arrival times having mean $\frac{1}{2}$ minute. The attendant takes an average of $\frac{1}{3}$ minute per car to supply petrol, the service times being exponentially distributed. Determine (i) the average number of cars waiting to be served, (ii) the average number of cars in the queue, and (iii) the proportion of time for which the pump attendant is idle.

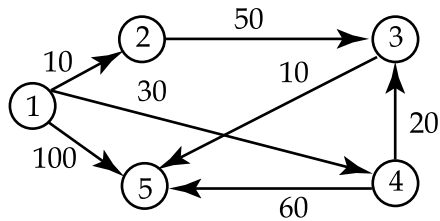
(10+8)

6. (a) A project has the following time schedule :

Activity	A	B	C	D	E	F	G
Preceding Activities	-	-	-	A, B	A, B	C, D, E	C, D, E
Duration (in days)	4	7	6	5	7	6	5

- (i) Draw the network and find the project completion time.
 (ii) Calculate total float for each activities.
 (iii) Critical path and its duration.

(b) Determine the shortest route from node 1 to node 5 using Dijkstra's algorithm :



(9+9)

7. (a) Find the optimum order quantity for a product for which the price breaks are as follows :

Quantity		$800 \leq Q_2$
Unit cost (Rs.)	1.00	0.98

The yearly demand for the product is 1600 units per year, cost of placing an order is Rs. 5, the cost of storage is 10% per year.

(b) Consider the following LPP :

$$\text{Max. } z = 5x_1 + 2x_2 + 3x_3$$

$$\text{s.t. } x_1 + 5x_2 + 2x_3 = 30; \quad x_1 - 5x_2 - 6x_3 \leq 40; \quad x_1, x_2, x_3 \geq 0.$$

(i) Write the associated dual problem.

(ii) Determine the associated optimal dual solution by solving primal problem using Big-M method.

(8+10)

- o o o -