

C0-R4.B1 : ELEMENTS OF MATHEMATICAL SCIENCES**NOTE :**

1. Answer question 1 and any FOUR from questions 2 to 7.
2. Parts of the same question should be answered together and in the same sequence.

Time : 3 Hours**Total Marks : 100**

1. (a) Using Cramer's rule, find the solution of system of simultaneous linear equations :
 $x - 7y = 3, -x + y = -9$
- (b) Evaluate the following :
 - (i) $\lim_{x \rightarrow 0} \frac{\sqrt[3]{1+x} - 1}{x}$
 - (ii) $\lim_{x \rightarrow 0} \frac{1 - \cos x}{x \sin x}$
- (c) Find all vertical and horizontal asymptotes of the family of curves :
 $\frac{a^2}{x^2} + \frac{b^2}{y^2} = 1$
- (d) Decide whether the following equations represent parabola, ellipse or hyperbola:
 - (i) $3x^2 - 18xy + 27y^2 - 5x + 7y + 4 = 0$
 - (ii) $2x^2 - \sqrt{15}xy + 2y^2 + x + y = 0$
- (e) Evaluate $\int_0^{\ln 2} \frac{2e^{4x} + 2e^{2x} - e^x}{e^{2x} + 1} dx$.
- (f) Check the convergence of the alternating series :

$$\sum_{n=1}^{\infty} \frac{\cos n\pi}{n^2 + 1}$$
- (g) If the sum of mean and variance of a Binomial distribution is 4.8 for 5 trials, find the probability of obtaining at least 4 successes. (7x4)
2. (a) Find the inverse of the following matrix by Gauss-Jordan Elimination method :

$$\begin{bmatrix} -1 & 1 & 2 \\ 3 & -1 & 1 \\ -1 & 3 & 4 \end{bmatrix}$$
- (b) Is the set of vectors $[1 \ 2 \ 4]$, $[2 \ -1 \ 3]$, $[0 \ 1 \ 2]$ and $[-3 \ 7 \ 2]$ linearly dependent? If so, find the relation between them.
- (c) Find the Eigen values and Eigen vectors of the matrix :

$$\begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix}$$
 (6+6+6)

3. (a) Sketch the graph of the function $y = \cos \frac{\pi x}{2}$. Also, find the period of the function.

(b) Find the values of λ so that the function

$$f(x) = \begin{cases} \frac{\lambda \cos x}{\pi - 2x}, & x \neq \pi/2 \\ 5, & x = \pi/2 \end{cases}$$

is continuous at $x = \pi/2$.

(c) Find the two points where the curve $x^2 + xy + y^2 = 7$ crosses the x -axis and show that the tangents to the curve at these points are parallel. What is the common slope of the tangents? Also, find the equations of the tangents at these points?

(d) Find the area of the region bounded by the curve $y = 2 - x^2$ and the line $y = -x$.

(4+4+5+5)

4. (a) Find a Cartesian equation of the hyperbola centered at origin that has a focus at $(3, 0)$ and the line $x = 1$ as a corresponding directrix.

(b) The position $P(x, y)$ of a particle moving in xy -plane is given by the equations and the parametric intervals

$$x = \sqrt{t}, y = t, t \geq 0$$

Identify the path traced by the particle and describe the motion.

(c) Find the Cartesian equations to the following Polar equations :

$$(i) \quad r^2 = 4r \cos \theta, \quad (ii) \quad r = \frac{4}{2 \cos \theta - \sin \theta}$$

(6+6+6)

5. (a) Discuss the convergence of the series :

$$\frac{1^2}{2^2} + \frac{1^2 \cdot 3^2}{2^2 \cdot 4^2} x + \frac{1^2 \cdot 3^2 \cdot 5^2}{2^2 \cdot 4^2 \cdot 6^2} x^2 + \dots \infty$$

(b) Expand $\ln(1 + \sin x)$ by Maclaurin's theorem.

(c) Estimate X when $Y = 10$, if the two lines of regression are $18X + Y = 18\lambda$ and $2X + Y = \mu$, where λ and μ being unknowns and mean of the distribution is at $(-1, 2)$. Also, find the values of λ , μ and coefficient of correlation.

(6+6+6)

6. (a) A petrol pump is supplied with petrol once a day. If its daily volume X of sales in thousands of litres is distributed by :

$$f(x) = 5(1-x)^4, 0 \leq x \leq 1$$

what must be the capacity of its tank in order that the probability that its supply will be exhausted in a given day shall be 0.01 ?

- (b) If X is uniformly distributed with mean 1 and variance $4/3$, find $P(X < 0)$.
- (c) If X is a Poisson variate such that $P(X=2) = 9xP(X=4) + 90xP(X=6)$. Find the mean value of X . Also, find $P(X > 0)$.

(6+6+6)

7. (a) In the two large populations, there are 30% and 25% respectively of blue-eyed people. Is the difference likely to be hidden in samples of 1200 and 900 respectively from the two populations ?

- (b) A die is thrown 60 times with the following results :

| | | | | | | |
|-----------|---|---|----|---|----|----|
| Face | 1 | 2 | 3 | 4 | 5 | 6 |
| Frequency | 8 | 7 | 12 | 8 | 14 | 11 |

Test at 5% level of significance if the die is honest, assuming that $P(\chi^2 > 11.1) = 0.05$ with 5 d.f.

(9+9)

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