No. of Printed Pages : **3**

Sl. No.

C3-R4 : MATHEMATICAL METHODS FOR COMPUTING

NOTE :

1. **Answer question 1 and any FOUR from questions 2 to 7.**
2. **Parts of the same question should be answered together and in the same sequence.**

Time : 3 Hours **Total Marks : 100**

1. (a) If $x_0 = 5$ and $x_n = 3x_{n-1}$ mod 150, find $x_1, x_2, \ldots, x_{10}$

(b) Let random variables X and Y have joint probability function $p(x_i, y_j) = \Pr\{X = x_i, Y = y_j\} = p_{ij}$, $i = 1, 2, \ldots, M$, $j = 1, 2, \ldots, L$. Show that $H(X, Y) = H(X) + H(Y|X)$.

(c) Consider the experiment of tossing two tetrahedra (regular four-sided polyhedron) each with sides labeled 1 to 4. Let X denote the number on the down turned face of the first tetrahedron and Y the larger of the down turned numbers. Find the joint probability mass function of (X, Y).

(d) Consider a communications system which transmits the digits 0 and 1. Each digit transmitted must pass through several stages, at each of which there is a probability $p = \frac{1}{3}$ that the digit that enters will be unchanged when it leaves. Suppose that the system consists of three stages. What is the probability that a digit entering the system as 0 will be transmitted by the second stage as 0 ?

(e) Consider an M/M/1 queue in which class 1 customers arrive at rate $\lambda_1 = 0.3$ and class 2 customers at rate $\lambda_2 = 0.5$ per unit time. Let the mean of the exponential service time be 1 time unit. Treating both classes identically results in a standard M/M/1 queue with $\lambda = 0.8$ and $\mu = 1$. Compute the mean number of customers in system and mean response time. Compute the mean number of class 1 customers and class 2 customers respectively when the non-preemptive priority policy is applied.

(f) Convert the following linear programming problem in standard form

$$\text{Max } Z = 3x_1 + 2x_2$$

Subject to

$$3x_1 + 4x_2 \le 5,$$

$$2x_1 + 6x_2 \ge 6,$$

$$x_1 + x_2 = 7,$$

$$x_1, x_2 \ge 0$$

(g) Determine the inverse Laplace transform $L^{-1}\left\{\dfrac{5s+1}{s^2 - s - 12}\right\}$

(7x4)

2. (a) Consider a discrete memoryless system X with symbols x_i, i = 1, 2, 3, 4 Following table list four possible binary codes.

x_i	**Code A**	**Code B**	**Code C**	**Code D**
x_1	00	0	0	0
x_2	01	10	11	100
x_3	10	11	100	110
x_4	11	110	110	111

Show that codes A and D are uniquely decodable but codes B and C are not uniquely decodable

(b) Show that the mutual information I(X; Y) of the channel with the input probabilities $p(x_i)$, i=1, 2,...,m, and the output probabilities $p(y_j)$, j=1, 2...,n,

can be expressed as : $\sum_{i=1}^{m} \sum_{j=1}^{n} p(x_i, y_j) \log_2 \frac{p(x_i \mid y_j)}{p(x_i)}$

(9+9)

3. (a) Consider a random process X(t) given by X(t)=Acos(ωt + Θ) where A and ω are constants and Θ is a uniform random variable over [−π, π]. Show that X(t) is ergodic in both the mean and the autocorrelation.

(b) Let (X, Y, Z) be a trivariate r.v. $f_{xyz}(x,y,z) = \begin{cases} ke^{-(ax+by+cz)} & x>0,\ y>0,\ z>0 \\ 0 & \text{otherwise} \end{cases}$

Where a, b, c>0 and k are constants.

(i) Determine the value of k.

(ii) Find the marginal joint pdf of X and Y.

(iii) Find the marginal pdf of X.

(iv) Find the marginal pdf of Y.

(9+9)

4. (a) Determine the Laplace transform of the half-wave rectifier output waveform defined by

$f(t) = 8\sin t, 0 < t < \pi$

$= 0, \pi < t < 2\pi, f(t+\pi) = f(t)$

(b) Determine the Fourier series for a periodic function defined by

$$f(t) = \begin{cases} 2(1+t) & -1 < t < 0 \\ 0 & 0 < t < 1 \end{cases}$$

$f(t+2) = f(t)$

(9+9)

5. (a) Two random processes X(t) and Y(t) are given by $X(t) = A\cos(\omega t + \Theta)$ $Y(t) = A\sin(\omega t + \Theta)$ where A and ω are constants and Θ is uniform over $(0, 2\pi)$. Find the cross-correlation function of X(t) and Y(t).

(b) Draw the state transitions diagram and classify the states of the Markov chain with following transition probability matrices.

(i) $P = \begin{bmatrix} 0 & 0.5 & 0.5 \\ 0.5 & 0 & 0.5 \\ 0.5 & 0.5 & 0 \end{bmatrix}$

(ii) $P = \begin{bmatrix} 0 & 0 & 0.5 & 0.5 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}$

(9+9)

6. (a) Consider a pure Markovian queueing system in which $\lambda_k = \begin{cases} \lambda & 0 \le k \le K \\ 2\lambda & K < k \end{cases}$

$\mu_k = \mu \;\; k = 1, 2, \ldots$
Find the equilibrium probabilities p_k for the number in the system.

(b) Solve the following linear programming problem using simplex method
Maximize $Z = 5x_1 + 2x_2$
Subject to $6x_1 + x_2 \geq 6$
$4x_1 + 3x_2 \geq 12$
$x_1 + 2x_2 \geq 4$
$x_1, x_2 \geq 0$

(9+9)

7. (a) Consider the following linear programming problem
Maximize $Z = 3x_1 + 5x_2$.
Subject to $x_1 \leq 4$
$2x_2 \leq 12$
$3x_1 + 2x_2 \leq 18$
$x_1, x_2 \geq 0$
Solve this problem using dynamic programming.

(b) Use the Kuhn-Tucker conditions, solve the problem :
Minimize $Z = (x_1 - 1)^2 + (x_2 - 2)^2$
Subject to $x_2 - x_1 = 1$
$x_1 + x_2 \leq 2$
$x_1 \geq 0, \; x_2 \geq 0$

(9+9)

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