B4.1-R4: COMPUTER BASED NUMERICAL AND STATISTICAL TECHNIQUES

NOTE:

1.	**Answer question 1 and any FOUR from questions 2 to 7.**
2.	**Parts of the same question should be answered together and in the same sequence.**

Time: 3 Hours **Total Marks: 100**

1.

a) If $u = 3v^7 - 6v$, find the relative maximum error in u at v = 1 if the error in v is 0.05.

b) A problem in Statistics is given to three students A, B and C whose chances of solving it are 0.50, 0.75 and 0.25 respectively. What is the probability that the problem will be solved by at least one of them?

c) Evaluate $\int_0^1 \frac{1}{1+x^2}dx$, correct to three decimal places by using Simpson's 1/3–rule with h = 0.25.

d) The profit of a new product is given by Z = 3 X – Y, X and Y are independent random variables with Var(X) = 1 and Var(Y) = 2. What is the variance of Z?

e) Using Newton-Raphson method, find a root of the equation

$$x^3 - 2x - 5 = 0$$

correct to three decimal places. (Take $x_0 = 3$)

f) The probability density function of a continuous random variable X is given by

$$f(x)=\begin{cases} kx \ ; 0 \le x \le 1 \\ k \ ; 1 \le x \le 2 \\ -kx + 3k \ ; 2 \le x \le 3 \\ 0 \ ; \text{elsewhere} \end{cases}$$

Determine the constant 'k' and compute $P(X \le 1.5)$.

g) The following data classify minor accidents over the past year at a certain industrial plant, according to the time periods in which these accidents occurred.

Time Period	Number of accidents
8-10 a.m.	47
10-12 p.m.	52
1-3 p.m.	57
3-5 p.m.	63

Test the hypothesis that each accident was equally likely to occur in any of the four time periods. Use the 5 percent level of significance.

(7x4)

2.

a) The probability density function of X is given by

$$f(x)=\begin{cases} a+bx^2, & 0\le x\le 1 \\ 0 & \text{otherwise} \end{cases}$$

If $E(X) = \frac{3}{5}$, find a and b.

b) The management of a restaurant that operates on reservations only, knows from experience that 15% of persons making table reservations will not show up. If the restaurant accepts 25 reservations, but has only 20 tables, what is the probability that all who show up will be accommodated?

c) One shot is fired from each of the three guns. E_1, E_2 and E_3 denote the events that the target is hit by the first, second and third guns respectively. If $P(E_1) = 0.5$, $P(E_2) = 0.6$ and $P(E_3) = 0.8$ and E_1, E_2 and E_3 are independent events, find the probability that exactly one hit is registered.

(6+6+6)

3.

a) If X and Y are independent Poisson random variables with respective mean λ_1 and λ_2, then calculate

i) $P\{X = k \mid X + Y = n\}$

ii) $E\{x \mid X + Y = n\}$

b) The probabilities, as estimated by a contractor, for the number of days required to complete a certain type of construction project are given below:

Time (days)	1	2	3	4	5
Probability	0.1	0.2	0.3	0.3	0.1

i) What is the expected time to complete a project?

ii) The contractor's project cost is made up of two parts – a fixed cost of Rs 10,000 plus Rs 1500 for each day taken to complete the project. What is the mean and standard deviation of total project cost?

(9+9)

4.

a) Suppose that X and Y are jointly distributed random variables. Show that

$Cov(aX, Y) = a\,Cov(X, Y)$

Further, show that

$$Cov\left(\sum_{i=1}^{n} X_i,\ \sum_{j=1}^{m} Y_j\right) = \sum_{i=1}^{n}\ \sum_{j=1}^{m} Cov(X_i, Y_j).$$

b) Regression of savings (y) of a family on income (x) may be expressed as

$y = a + (1/m)\,x$

where, 'a' and 'm' are constants. It is given that the variance of savings is one quarter of the variance of incomes and the correlation coefficient is found to be 0.4. Calculate the value of 'm'.

(9+9)

5.

a) From a partially destroyed laboratory only following records could be made available: $x = 4y + 5$ and $y = kx + 4$ are the regression lines of x on y and y on x respectively. Show that –

i) $0 \le k \le 1/4$

ii) If $k = 1/16$, find the means of two variables and the coefficient of correlation.

b) The local municipality in a certain town installed 2,000 electric lamps in streets and parks of the town. The lamps have an average life of 1,000 burning hours with standard deviation of 200 hours. Assuming the lives of lamps are normally distributed, calculate the following:

i) How many lamps are expected to fuse in the first 700 burning hours?

ii) After how many hours of burning, 10% of the lamps are expected to fuse?

Given that $P(z \le 1.5) = 0.933$ and $P(z \ge 1.28) = 0.1$, where 'z' is a standard normal variable.

(9+9)

6.

a) Given that f(0) = 1, f(1) = 0, f(2) = 1 and f(3) = 10. Using Newton's Forward Difference interpolation formula, find the cubic polynomial which fits the given data. Hence, or otherwise, compute the value f(4).

b) Solve the equations by the Gauss-Seidel Method–

$$10x - 2y - z - w = 3$$
$$-2x + 10y - z - w = 15$$
$$-x - y + 10z - 2w = 27$$
$$-x - y - 2z + 10w = -9$$

(8+10)

7.

a) The manager of computer operations of a large company wants to study computer usage of two departments within the company – the accounting department and the research department. A random sample of five jobs from the accounting department and six jobs from the research department in the past week are selected and the processing time (in seconds) for each job is recorded. The same are presented below:

Accounting	9	3	8	7	12	
Research	4	13	10	9	9	6

i) Is there evidence to conclude that the mean processing time in the research department is six seconds against the alternative that it is more than six seconds? Compute p-value and draw inferences at 10% level of significance. Given that $P(t_{5d.f.} \geq 1.9462) = 0.0546$.

ii) Is there evidence of a significant difference between the mean processing time of the accounting department and that of research department? (Assume $\alpha = 0.10$).
Given that $P[|t| \geq 1.8331] = 0.10$

b) A newly launched mineral water company wants to know whether the demand for the number of mineral water bottles is the same for each day. The company collected data in terms of the number of bottles sold per day from a randomly selected departmental store. Data are presented in the Table given below:

Days	Mon	Tue	Wed	Thurs	Fri	Sat	Sun
Number of bottles sold	140	160	136	175	125	178	178

Test the hypothesis at 5% level of significance that the demand for mineral water bottles is same for all the days.
Given that $P[\chi^2(6 \text{ d.f.}) \geq 12.6] = 0.05$

(9+9)