No. of Printed Pages : **3**

Sl. No.

B0-R4 : BASIC MATHEMATICS

[redacted]

Time : 3 Hours **Total Marks : 100**

1. (a) Reduce $1-\cos\alpha+i\sin\alpha$ to the modulus and amplitude form.

(b) If $A=\begin{bmatrix}1 & 2\\ 4 & -3\end{bmatrix}$ and $f(x)=2x^2-4x+5$, then find the value of f (A).

(c) If 1, ω, ω^2 are the cube roots of the unity, then find the value of $\begin{vmatrix}1 & \omega^n & \omega^{2n}\\ \omega^{2n} & 1 & \omega^n\\ \omega^n & \omega^{2n} & 1\end{vmatrix}$.

(d) Find the value of $\int e^{-\ln x}dx$.

(e) Find the value of $\underset{n\to\infty}{Limit}\left[\frac{4^n - 3^n}{4^n + 3^n}\right]$.

(f) Solve the differential equation $\frac{dy}{dx} = \frac{1+y^2}{1+x^2}$.

(g) Find a parametric equation for the line which passes through the point (5, 2, 4) and parallel to the vector $v=4\hat{i}+7\hat{j}-9\hat{k}$.

(4x7)

2. (a) Find the real values of x and y so that $-3+i\,x^2\,y$ and x^2+y+4i may represent complex conjugate numbers.

(b) If z_1, z_2, z_3 be the vertices of an isosceles triangle, right angled at z_2, prove that $z_1^2+z_3^2+2z_2^2=2z_2(z_1+z_3)$.

(c) Show that the roots of the equation $(x-1)^n=x^n$, n being a positive integer are $\frac{1}{2}\left(1+i\cot\frac{r\pi}{n}\right)$, where r has the values 1, 2, 3,..., $n-1$.

(d) Find the values of λ for which the equations

$$(\lambda-1)x+(3\lambda+1)y+2\lambda z=0$$
$$(\lambda-1)x+(4\lambda-2)y+(\lambda+3)z=0$$
$$2x+(3\lambda+1)y+3(\lambda-1)z=0$$

are consistent, and find the ratios of $x:y:z$ when λ has the smallest of these values.

(3+5+5+5)

3. (a) Find the characteristic equation of the matrix $A = \begin{vmatrix} 2 & 1 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 2 \end{vmatrix}$ and hence find the matrix represented by $A^8 - 5A^7 + 7A^6 - 3A^5 + A^4 - 5A^3 + 8A^2 - 2A + I$.

(b) Find the value of $\underset{x \to 0}{Limit} \dfrac{x\left(1 - \sqrt{1 - x^2}\right)}{\sqrt{1 - x^2}\,(\sin^{-1} x)^3}$.

(c) Find the value of the derivative of $f(x) = |x-1| + |x-3|$ at $x = 2$.

(d) If $x\sqrt{1+y} + y\sqrt{1+x} = 0$, then find the value of $\dfrac{dy}{dx}$ in terms of x.

(5+4+4+5)

4. (a) Find the volume of the largest possible right-circular cylinder that can be inscribed in a sphere of radius a.

(b) Find the asymptotes of the curve $x^3 + 3x^2y - 4y^3 - x + y + 3 = 0$.

(c) Evaluate $\int_0^{\pi} \cos 2x . \log(\sin x)\, dx$.

(d) Find the area common to the parabola $y^2 = ax$ and the circle $x^2 + y^2 = 4ax$.

(e) Find the length of the arc of the parabola $x^2 = 4ay$ measured from the vertex to one extremity of the latus rectum.

(4+4+3+4+3)

5. (a) Find the volume formed by the revolution of the loop of the curve $y^2(a+x) = x^2(3a-x)$ about the x-axis.

(b) Test the convergence of the following series.

(i) $\dfrac{1}{2\sqrt{1}} + \dfrac{x^2}{3\sqrt{2}} + \dfrac{x^4}{4\sqrt{3}} + \dfrac{x^6}{5\sqrt{4}} + ...$

(ii) $\dfrac{1}{\log 2} - \dfrac{1}{\log 3} + \dfrac{1}{\log 4} - \dfrac{1}{\log 5} + ...$

(c) Write the statement of Integral Test and use it to show that the series $\sum_{n=1}^{\infty} \dfrac{1}{n^p}$ is convergent if $p > 1$ and divergent if $p \le 1$.

(d) Expand $\log_e x$ in powers of $(x-1)$.

(5+[3+3]+4+3)

6. (a) Find the solution of the boundary value problem $x\frac{dy}{dx} + y = y^2 \log x,\ y(1) = 1.$

(b) Find the solution of the following differential equations :

(i) $\left[\frac{d^2y}{dx^2} + y\right]^3 = 0.$

(ii) $\frac{d^2x}{dt^2} + 6\frac{dx}{dt} + 9x = 0.$

(c) Find the length of the curve $x = t\sin t$, $y = t\cos t$ from $t = 0$ to $t = 2\pi$.

(d) Find the equation of the tangent to the curve $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ at point $(a\cos\theta, b\sin\theta)$.

(4+[3+3]+4+4)

7. (a) The focal distance of a point on the parabola $y^2 = 12\,x$ is 4. Find the abscissa of the point.

(b) If the angle between the lines joining the foci of an ellipse to an extremity of the minor axis is 90°, then find the eccentricity of the ellipse.

(c) If $|\vec{a}| = 2, |\vec{b}| = 7$ and $\vec{a} \times \vec{b} = 3\hat{i} + 2\hat{j} + 6\hat{k}$, then find the angle between $\vec{a}$ and $\vec{b}$.

(d) Let $\vec{a} = 4\hat{i} + 3\hat{j} + \hat{k},\ \vec{b} = 2\hat{i} - \hat{j} + 2\hat{k}$. Then find a unit vector $\hat{n}$ which is perpendicular to vectors $\vec{a}$ and $\vec{b}$ both.

(e) Find the equation of the plane which passes through the point (3, −3, 1) and is perpendicular to the planes $7x + y + 2z = 6$ and $3x + 5y - 6z = 8$.

(4+4+3+3+4)

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