No. of Printed Pages : **3**

Sl. No.

B0.1-R5 : BASIC MATHEMATICS

NOTE :

1. Answer question 1 and any FOUR questions from 2 to 7.

2. Parts of the same question should be answered together and in the same sequence.

Time : 3 Hours **Total Marks : 100**

1. (a) Examine the convergence of the series $2-\frac{3}{2}+\frac{4}{3}-\frac{5}{4}+\ldots.$ **(3)**

(b) Find the rank of the matrix $\begin{bmatrix} 1 & 1 & 2 \\ 1 & 2 & 3 \\ 0 & -1 & -1 \end{bmatrix}$. **(3)**

(c) Solve the following system of simultaneous linear equations by Cramer's rule : **(3)**

$$2x-3y=2$$
$$3x-2y=8$$

(d) If a point $P(a, b)$ lies on the line $3x+2y=13$ and another point Q(b, a) lies on the line $4x-y=5$, then find the equation of straight line PQ. **(3)**

(e) Find the domain and range of the function $f(x)=\sqrt{16-25x^2}$. **(3)**

(f) Find the projection of the vector $\hat{i}+3\hat{j}-2\hat{k}$ on the vector $2\hat{i}-6\hat{j}+3\hat{k}$. **(3)**

(g) Find the angle of intersection of the curve $x^2=2y$ and $y^2=16x$. **(4)**

(h) Evaluate $\int_0^1 \frac{\sin^{-1} x}{x}\, dx$. **(3)**

(i) Find the area of triangle whose vertices are A(1, 2, 3), B(2, -1, 4) and C(4, 5, -1). **(3)**

2. (a) Discuss the convergence of the series $\sum_{n=2}^{\infty} \frac{1}{n(\log_e n)^p}$, (p > 0) **(6)**

(b) Discuss the convergence of the series $1+\frac{2!}{2^2}+\frac{3!}{3^3}+\frac{4!}{4^4}+\ldots$ **(4)**

(c) Find the parametric and symmetric equation of the line that passes through the point (1, -3, 2) in the direction of the vector $\vec{v}=\hat{j}-2\hat{k}$. **(4)**

(d) Find the equation of the plane which passes through the point (3, -3, 1) and is perpendicular to the planes $7x+y+2z=6$ and $3x+5y-6z=8$. **(4)**

3. (a) Find the Eigen values and Eigen vectors of the matrix $\begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$. **(6)**

(b) Find the values of p for which the system of simultaneous linear equations

$$(p-1)x+(3p+1)y+2pz=0$$
$$(p-1)x+(4p-2)y+(p+3)z=0$$
$$2x+(3p+1)y+3(p-1)z=0$$

has non-trivial solution. Also find the ratio $x : y : z$ when p has the smallest of these values. **(6)**

(c) Prove that $\begin{vmatrix} 1+a^2-b^2 & 2ab & -2b \\ 2ab & 1-a^2+b^2 & 2a \\ 2b & -2a & 1-a^2-b^2 \end{vmatrix}$ is a perfect cube. **(6)**

4. (a) Show that the equation $2x^2-13xy-7y^2+x+23y-6=0$ represents a pair of straight lines. Also find the angle between them and the co-ordinates of the point of intersection of the lines. **(6)**

(b) Find the equation of the parabola whose co-ordinates of vertex and focus are $(-2, 3)$ and $(1, 3)$ respectively. **(4)**

(c) Find the equations of tangent and normal to the circle $x^2+y^2-3x+4y-19=0$ at the point (2, 3). **(4)**

(d) Graph the ellipse $\frac{(x+2)^2}{4}+\frac{(y-5)^2}{9}=1$. Also, label the centre, vertices and foci. **(4)**

5. (a) Using Maclaurin's Theorem, prove that $\ln(1+e^x) = \ln 2 + \frac{x}{2} + \frac{x^2}{8} - \frac{x^4}{192} + \ldots$ **(6)**

(b) If $\ln y = a\cos^{-1} x$, show that $(1-x^2)y_{n+2}-(2n+1)xy_{n+1}=(a^2+n^2)y_n$. **(6)**

(c) If $0 < a < b < 1$, using Mean value theorem, prove that $\frac{b-a}{1+b^2} < \tan^{-1}b - \tan^{-1}a < \frac{b-a}{1+a^2}$. Hence show that $\frac{\pi}{4}+\frac{3}{25} < \tan^{-1}\frac{4}{3} < \frac{\pi}{4}+\frac{1}{6}$. **(6)**

6. (a) Find all the asymptotes of the family of curve $(x+y)^2(x+y+2)=x+y+2$. **(5)**

(b) Show that the right circular cylinder of given surface including the ends and maximum volume is such that the radius of its base is half of its height. **(5)**

(c) Find the value of **a** if the function

$$f(x) = \begin{cases} \dfrac{\tan x - \cot x}{x - \frac{\pi}{4}}, & x \neq \dfrac{\pi}{4} \\ \text{a}, & x = \dfrac{\pi}{4} \end{cases}$$

is continuous at $x = \dfrac{\pi}{4}$. **(4)**

(d) Find the area between the parabola $y^2 = 4\text{a}x$ and the line $x + y = 3\text{a}$. **(4)**

7. (a) Evaluate $\displaystyle\int_0^1 \frac{\ln(1+x)}{1+x^2}dx$. **(3)**

(b) Evaluate $\displaystyle\int_0^\infty \frac{1}{(x^2+\text{a}^2)(x^2+\text{b}^2)}dx$. **(3)**

(c) If n is an odd natural number, prove that $\displaystyle\int_0^\pi \frac{x \sin nx}{\sin x}dx = \frac{\pi^2}{2}$ **(6)**

(d) Trace the curve $y^2(2\text{a}-x)=x^3$ and find the area between the curve and its asymptote. **(6)**

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