No. of Printed Pages : **4**

Sl. No.

BE10-R4 : APPLIED OPERATIONS RESEARCH

NOTE :

1. **Answer question 1 and any FOUR from questions 2 to 7.**
2. **Parts of the same question should be answered together and in the same sequence.**

Time : 3 Hours **Total Marks : 100**

1. (a) A firm makes products x and y and has a total production capacity of 9 tons per day, x and y requiring the same production capacity. The firm has a permanent contract to supply at least 2 tons of x and at least 3 tons of y per day to another company. Each ton of x requires 1 machine hours production time and each ton of y requires 2 machine hours production time and the daily maximum possible number of machine hours is 12. All the firms output can be sold, and the profit made is Rs. 80 per ton of x and Rs. 120 per ton of y. It is required to determine the production schedule for maximum profit. Formulate the linear programming problem

(b) Solve the following LPP without using any artificial variable.

Max. $z = 2x_1 + 4x_2 + 4x_3 - 3x_4$

S.t. $x_1 + x_2 + x_3 = 4$; $x_1 + 4x_2 + x_4 = 8$; $x_1, x_2, x_3, x_4 \geqslant 0$.

(c) In a zero-sum game of tossing and matching two coins with two players, suppose A wins one unit of value, when there are two heads, wins nothing when there are two tails and loses ½ unit of value when there is one head and one tail. Determine the payoff matrix of player A and the best strategies for each of the two players.

(d) In a 3×3 transportation problem, let x_{ij} be the amount shipped from source i to destination j and C_{ij} be the corresponding transportation cost per unit. The amounts of supply at sources 1, 2 and 3 are 15, 30 and 85 units, respectively; and the demands at destinations 1, 2 and 3 are 20, 30 and 80 units respectively. Assume that starting with north west-corner method the solution and that the associated values of multipliers are given as $u_1 = -2$, $u_2 = 3$, $u_3 = 5$, $v_1 = 2$, $v_2 = 5$ and $v_3 = 10$. Find the Corresponding solution x_{ij} and the associated cost.

(e) The demand for an item in a company is 18000 units per year and the company can produce the items at a rate of 3000 per month. The cost of one set-up is Rs. 500/- and the holding cost of 1 unit per month is 15 paise. The shortage cost of one unit is Rs. 20/- per month. Determine optimum production batch quantity.

(f) Arrivals at a telephone booth are considered to be Poisson, with an average time of 10 minutes between the arrivals. The length of a phone call is assumed to be exponentially distributed with mean 3 minutes. What is the probability that a person arriving at the booth will have to wait and what is the average length of the queue ?

(g) Draw the network of the project for which (i) activities A and B start simultaneously, (ii) activities A, C, B, F precede C, D, E, H respectively, (iii) E preceded F and G, (iv) D,G,H precede I, (v) I is a terminal activity.

(7 x 4)

2. (a) A toll road has only one drive- in window. Cars arrive according to Poisson distribution at the rate of 2 cars every 5 minutes. The space in front of the window can accommodate at most 10 cars including the one being served. Other cars can wait outside this space if necessary. The service time per customer is exponential with mean of 1.5 minutes. Determine.

(i) the probability that the facility is idle.

(ii) the expected waiting time until a customer reaches the window.

(iii) the expected number of customers waiting to be served.

(iv) the probability that the waiting line will exceed the 10 space capacity.

(b) Perform two complete iteration of the Steepest Descent method to find the minimum value of the function $f(x_1, x_2) = 2x_1^2 + x_2^2$ starting from the initial point $(x_1, x_2) = (1, 2)$.

(9+9)

3. (a) A and B play game in which each has three coins 5p, 10p and a 20p. Each selects a coin without the knowledge of the other person choice. If the sum of the coins is an odd amount, then A wins B's coin. If the sum on coins is even, then B wins A's coin. Find the optimal strategy for each player and the value of the game, using the principle of dominance.

(b) Consider the following LPP :

Max. $z = x_1 + 5x_2 + 3x_3$

S.t. $x_1 + 2x_2 + x_3 = 3;\ 2x_1 - x_2 = 4;\ x_1, x_2, x_3 \geq 0.$

(i) Write the associated dual problem.

(ii) Given that the optimal basic variables are x_1 and x_3, determine the associated optimal dual solution.

(10+8)

4. (a) The demand of an item is uniform by distributed at a rate of 20 units per month. The fixed cost is Rs.10 each time a production run is made. The production cost is Rs. 1 per item, and the inventory carrying cost is Rs. 0.25 per item per month. If the shortage cost is Rs. 1.25 per item per month, determine how often to make a production run and what must be the optimal production size ?

(b) A company has six jobs on hand coded 'A' to 'F'. All the jobs have to do through two machines M1 and M2. The time required for each job on each machine, in hours, is given below :

	A	B	C	D	E	F
M1	3	12	18	9	15	6
M2	9	18	24	24	3	15

Find the optimum sequence of jobs that minimizes the total elapsed time to complete the jobs. Also compute the idle time for both the machines.

(8+10)

5. (a) A company has three plants and four warehouses. It is given the supply for plants A, B and C are 10, 25, 20 units respectively, and the demand in units at warehouses I, II, III, IV are 25, 10, 15, 5 units respectively. The corresponding transportation costs are given in the following table :

Warehouses

Plants		I	II	III	IV
	A	5	10	4	5
Plants	B	6	8	7	2
	C	4	2	5	7

(i) Is this solution feasible? (ii) Is this solution optimal? (iii) If the cost for the route B-III is reduced from Rs. 7 to Rs. 6 per unit, what will be the optimum solution?

(b) A firm employs typists on hourly piece-rate basis for daily work. There are four typists and their charges and speed are different. It has been agreed that only one job will be given to one typist and the typist is paid for a full hour even when he works for a fraction of an hour. Find the least cost allocation for the following data :

Typist	**Rate/hour (Rs.)**	**Number of pages typed/hour**	**Job**	**No. of pages**
A	4	8	P	102
B	3	10	Q	135
C	5	11	R	110
D	3	9	S	85

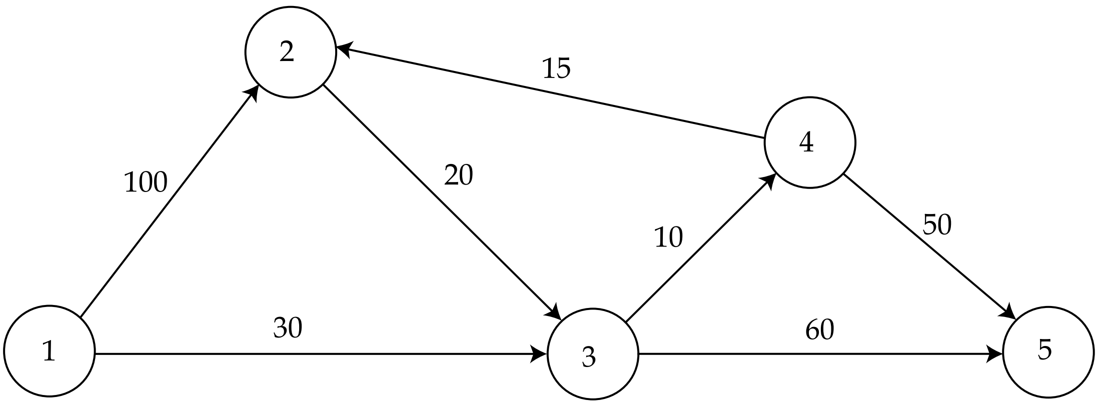
(10+8)

6. (a) Develop the Branch and bound tree for the following problems. Use x_1 as the branching variable at node 0.

Max. $z = x_1 + x_2$

s.t. $2x_1 + 5x_2 \leq 16$; $6x_1 + 5x_2 \leq 27$; $x_1, x_2 \geq 0$ and both are integers.

(b) The network in the following figures gives the permissible routes and their lengths in miles between city 1 (node1) and four other cities (nodes 2 to 5). Determine the shortest routes from city 1 to city 5 using Dijkastra's algorithm. Explain the steps clearly.



(9+9)

7. (a) Using two-phase method solve the following LPP :

Max. $z = 2x_1 + 2x_2 + 4x_3$

S.t. $2x_1 + x_2 + x_3 \leq 2$; $3x_1 + 4x_2 + 3x_3 \geq 8$; $x_1, x_2, x_3 \geq 0$

(b) The following table gives the activities in a construction project and time duration :

Activity	1-2	1-3	2-3	2-4	3-4	4-5
Preceding activity	---	---	1-2	1-2	1-3, 2-3	2-4, 3-4
Normal time (days)	20	25	10	12	5	10

(i) Draw the activity network of the project.

(ii) Determine the critical path and the project duration.

(9+9)

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