C0-R4.B1 : ELEMENTS OF MATHEMATICAL SCIENCES

NOTE :
1. Answer question 1 and any FOUR questions from 2 to 7.
2. Parts of the same question should be answered together and in the same sequence.

Total Time : 3 Hours   **Total Marks : 100**

1. (a) If $A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$. Prove that $(2A + 3B)^3 = 8A + 36B$.

     (b) Find $\dfrac{dy}{dx}$ if $y = (\log_e x)^{\sin x}$.

     (c) Evaluate $\displaystyle\int_{0}^{\frac{\pi}{2}} x.\sin(x)dx$

     (d) Verify the mean value theorem for $f(x) = \log_e x$ in $[1, e]$.

     (e) If vector $\overrightarrow{AB} = 5\hat{i} + 2\hat{j} - 6\hat{k}$ and the coordinate of A are (1, 2, 2), find the coordinates of B.

     (f) One card is drawn at random from a pack of well - shuffled deck of cards.
         Let E : The card drawn is spade.
             F : The card drawn is an ace.
         Are the event E and F are independent ?

     (g) Find the Eigen values of the matrix

         $A = \begin{bmatrix} 4 & 2 \\ 3 & 3 \end{bmatrix}$ **(7x4)**

2. (a) Show that the function defined by,

         $f(x) = \begin{cases} (x^2 - 9)/(x - 3), & x \neq 3 \\ 6, & x = 3 \end{cases}$ is continuous at $x = 3$.

     (b) Find the rank of the matrix.

         $\begin{bmatrix} 1 & 3 & 4 \\ 2 & 4 & 6 \\ 3 & 5 & 9 \end{bmatrix}$

     (c) Find $\dfrac{dy}{dx}$ of following.

         (i) $x^2 + y^2 = a^2$ (ii) $xy - \log_e(xy) = \log_e 2$.   **(4+5+9)**

3. (a) Find mean and variance for the number of tails when a coin is tossed thrice.

(b) Solve the equation $\begin{vmatrix} 3x-8 & 3 & 3 \\ 3 & 3x-8 & 3 \\ 3 & 3 & 3x-8 \end{vmatrix} = 0$

(c) Evaluate : $\int \dfrac{3x}{(x-1)(x-2)(x-3)}\, dx$

(6+6+6)

4. (a) Check whether the system of equations is consistent or not. If consistent find the solution.

$$x + y + z = 3$$
$$x + 2y + 3z = 4$$
$$x + 4y + 9z = 6$$

(b) Evaluate : $\lim\limits_{x \to 0} (x)^x$

(c) Using the method of integration, find the area bounded by the curves $|x| + |y| = 1$.

(6+6+6)

5. (a) Discuss the convergence of following series.

$$1 + \dfrac{2!}{2^2} + \dfrac{3!}{3^3} + \dfrac{4!}{4^4} + \dfrac{5!}{5^5} + \ldots \infty$$

(b) The following table shows the sales and expenditure of a company.

	Sale	Expenditure (Rs. Crores)
Mean	40	6
SD	10	1.5

If the Coefficient of correlation $r = 0.9$. Estimate the likely sales for a proposed expenditure of Rs. 10 crores.

(c) A machine makes a product's component with a 1.6 cm standard deviation in length. From the output, a 64-component random sample was taken, with a mean length of 90 cm. If the part is either less than 88 cm or more than 92 cm, the customer will refuse it. Does the 95% confidence interval guarantee that the customer will take all of the produced components ? The significant value at 5% is $Z_{a/2} = 1.96$.

(6+6+6)

6. (a) Use Cramer's rule to solve the following system of equations.
$3x + y - z = 1$, $5x + 2y + 3z = 2$, $8x + 3y + z = 3$

(b) Find the radius and center of the circle $x^2 + y^2 - 4x - 8y - 45 = 0$

(c) Find the angle between vectors $\vec{a} = 2\hat{i} + 2\hat{j} - \hat{k}$ and $\vec{b} = 6\hat{i} - 3\hat{j} + 2\hat{k}$.

(6+6+6)

7. (a) A speak the truth 8 times out of 10 times. He tossed a die. He report that it was 5. What is the probability that it was actually 5.

(b) Prove that $\int_0^{\frac{\pi}{2}} \sin 2x . \log_e(\tan x).dx = 0$

(c) Test the convergence of series.

$$\frac{1}{1.2.3} + \frac{3}{2.3.4} + \frac{5}{3.4.5} + \dots \infty$$

(6+6+6)

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