No. of Printed Pages : **4**

Sl. No.

C3-R4 : MATHEMATICAL METHODS FOR COMPUTING

NOTE :

1. **Answer question 1 and any FOUR questions from 2 to 7.**
2. **Parts of the same questions should be answered together and in the same sequence.**

Total Time : 3 Hours **Total Marks : 100**

1. (a) A bowl contains 16 chips, of which 6 are red, 7 are white, and 3 are blue. If four chips are taken at random and without replacement, find the probability that :

(i) each of the four chips is red;

(ii) none of the four chips is red;

(iii) there is at least one chip of each color.

(b) The joint pdf between stream flows in two rivers is given by

$$f(x, y) = x^2 + \frac{xy}{3} \quad 0 \le x \le 1; 0 \le y \le 2$$

Determine $P(X \geqslant 1/2)$

(c) Suppose that the probability of a dry day following a rainy day is 1/3 and that the probability of a rainy day following a dry day is 1/2. Given that May 1 is a dry day. Find the probability that May 3 is also a dry day.

(d) Suppose the people arrive in a bank at a Poisson rate of one per every 4 minutes and that the service time is exponential at a rate of one service per 3 minutes. What is

(i) the average number of people in the bank, and

(ii) the average time a person spends in the bank ?

(e) Find f(x) for which the Laplace transform is : $\mathcal{L}(f(x)) = F(s) = \frac{s}{(s^2+16)^2}$

(f) Solve the following using Simplex method :

Min $Z = -2x_1 = -3x_2$ subjected to the constraints :

$$2x_1 + x_2 \le 4$$

$$x_1 + 2x_2 \le 5,$$

$$x_1, x_2 \geqslant 0$$

(g) Let (X, Y) have the following joint distribution :

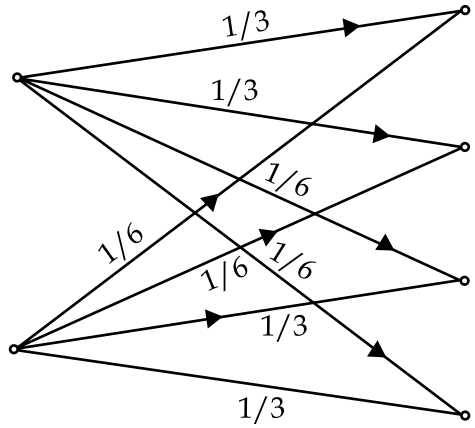
Y \ X	1	2	3	4
1	$\frac{1}{8}$	$\frac{1}{16}$	$\frac{1}{32}$	$\frac{1}{32}$
2	$\frac{1}{16}$	$\frac{1}{8}$	$\frac{1}{32}$	$\frac{1}{32}$
3	$\frac{1}{16}$	$\frac{1}{16}$	$\frac{1}{16}$	$\frac{1}{16}$
4	$\frac{1}{4}$	0	0	0

find H(X), H(Y), and H(X|Y).

(7x4)

2. (a) Let X_1 and X_2 have joined pdf
$f(x_1, x_2) = 2$ if $0 < x_1 < x_2 < 1$, and $= 0$ elsewhere
Find conditional expectation $E(X_2|X_1=x_1)$, and $E(X_1|X_2=x_2)$.

(b) Find the capacity of the channel given by



(10+8)

3. (a) Let $p(x)$, $q(x)$, $x \in \chi$ (range set of a random variable, X), be two probability mass function. Then show that $D(p||q) \geq 0$, with equality if and only of $p(x) = q(x)$ for all x.

(b) Solve the following LPP by Simplex method :
Max $Z = 2x_1 + x_2$ subject to
$3x_1 + 5x_2 \leq 15$
$6x_1 + 2x_2 \leq 24$,
$x_1, x_2 \geq 0$.

(9+9)

4. (a) Use the Laplace transform to solve the following :

$$y' + 4\int_0^t y(s)ds = 6, y(0) = 1$$

(b) Consider the function

$$f(t)=\begin{cases} t, & if \quad t\in[0,\ 1) \\ 1, & if \quad t\in[1,\ 2] \end{cases}$$

Find its Fourier series and determine its sum.

(9+9)

5. (a) Patients arrive at a doctor's clinic according to Poisson distribution at a rate of 30 patients per hour. The waiting room does not accommodate more than 9 patients. Examination time per patient is exponential with mean rate of 20 per hour. Find the

(i) probability that an arriving patient will not wait,

(ii) effective arrival rate,

(iii) average number of patients in the clinic,

(iv) average number of patients in a queue.

(b) A petrol pump station has 4 pumps. The service times follow the exponential distribution with a mean of 6 min and cars arrive for service in a Poisson process at the rate of 30 cars per hour.

(i) What is the probability that an arrival would have to wait in line ?

(ii) Find the average waiting time, average time spent in the system and the average number of cars in the system.

(9+9)

6. (a) If customers arrive at a counter in accordance with a Poisson process with a mean rate of 2 per minute, find the probability that the interval between two consecutive arrivals is

(i) more than 1 minute

(ii) between 1 minute and 2 minutes

(iii) less than 4 or equal to 4 minutes

(b) Draw the state diagram of a birth and death process and obtain the balance equations.

(9+9)

7. (a) Use the branch and bound algorithm method to solve the following Integer Linear Programming problem :

$$\text{Max } Z = x_1 + 2x_2$$

such that

$$2x_1 + x_2 \leq 7$$

$$-x_1 + x_2 \leq 3,$$

$x_1, x_2 \geq 0$; x_1, x_2 are positive integers

(b) Solve the following non-linear problem

$$\text{Min } (1 - x^2) \text{ subjected to}$$

$$-(x+1)^3 \leq 0$$

$$(x-2) \leq 0$$

(10+8)

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