No. of Printed Pages : **3**

Sl. No.

C4-R4 : ADVANCED ALGORITHMS

NOTE :

1. Answer question 1 and any FOUR questions from 2 to 7.

2. Parts of the same question should be answered together and in the same sequence.

Total Time : 3 Hours **Total Marks : 100**

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1. (a) Explain the various properties of an algorithm with one example.

(b) Consider the following search problem :

Input : A sequence $A = < a_1, a_2 \ldots, a_n >$ and a value of v.

Output : An index i if $v = A[i]$ for $i = 1$ to n, otherwise return NIL.

Write pseudo code for the linear search, which scans the given sequence A, looking for v. using a loop invariant; prove that your algorithm is correct. Make sure that your loop invariant fulfils the three conditions.

(c) Define different design approaches. Discuss why we should use branch and bound algorithm ?

(d) Prove that $n! = O(n^n)$.

(e) What is Flynn's Classification ? Using suitable example discuss each of them.

(f) Write a short note on the following :

(i) NP - hard problem.

(ii) NP - complete problem.

(g) "Banks often record transaction on an account in order of the times of the transactions, but many people like to receive their bank statements with checks listed in order by check number. People usually write check in order by check number, and merchants usually cash them with reasonable dispatch. The problem of converting time-of-transaction ordering to check-number ordering is therefore the problem of sorting almost-sorted input. Argue that the procedure INSERTION-SORT would tend to beat the procedure QUICK-SORT on this problem."

(7x4)

2. (a) Compare the Naïve string matching and Boyer - Moore string matching Algorithm.

(b) What is an optimal Huffman code for the following set of frequencies :

a : 45, b : 13, c : 12, d : 16, e : 9 , f : 5.

(c) Compute the prefix function π for the pattern ababbabbabbababbab where the alphabet is $\Sigma = (a, b)$.

(6+8+4)

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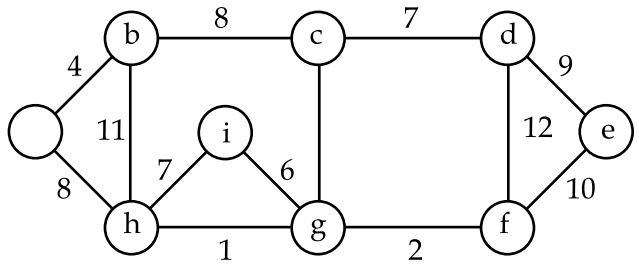
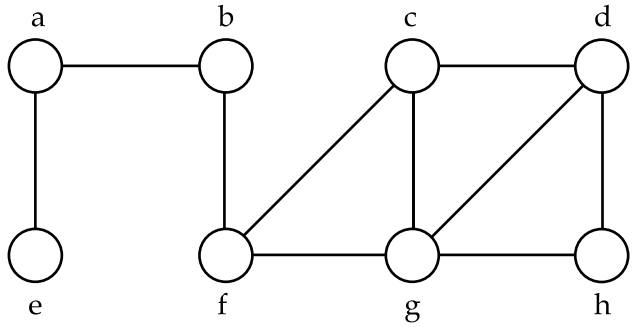
3. (a) Find the minimum spanning tree for the following graph using Prim's algorithm.



(b) Illustrate the HEAP - EXTRACT - MAX operation on the following Max-Heap.
A = <28, 25, 10, 5, 12, 6, 7, 4, 0, 11, 2, 1>

(c) Consider the graph G shown in the Fig. Considering starting Vertex as 'd', illustrate the steps of Breadth First Search (BFS).



(6+6+6)

4. (a) Given an adjacency - list representation for a complete binary tree on 7 vertices. Give an equivalent adjacency - matrix representation. Assume that vertices are numbered level wise from 1 to 7 i.e.at level 0, node number is 1, at level 1, node numbers are 2 (left child of 1) and 3(right child of 1) and so on.

(b) A sequence of n operations is performed on a data structure. The i^{th} operation costs i if i is an exact power of 2 otherwise 1. Use aggregate analysis to determine the amortized cost per operation.

(c) Write the recurrence relation for Tower of Hanoi problem and solve it.

(4+8+6)

5. (a) Differentiate between Prim's algorithm and Kruskal's algorithm in terms of the construction of the MST tree.

(b) Consider 5 items along with their respective weights and values :
$I = <I_1, I_2, I_3, I_4, I_5>$
w = <5, 10, 15, 20, 30>
v = <50, 20, 60, 100, 210>
The capacity of knapsack W = 50. Find the solution to the fractional knapsack problem.

(c) Give an $O(n^2)$ time algorithm to find the longest monotonically increasing subsequence of a sequence of n numbers.

(4+8+6)

6. (a) Illustrate the operations of Bubble sort on the array A = <5, 2, 1, 4, 3, 8, 7>.

(b) Insertion sort can be expressed as a recursive procedure: recursively sort A[1 …n-1], then insert A[n] into the sorted array. Write a recurrence for the running time of this recursive version.

(c) Describe the following :

(i) Best-case time for Quick Sort.

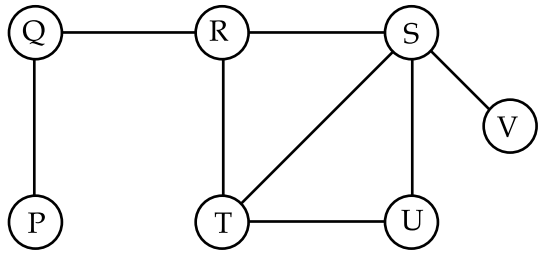
(ii) Worst - case time for Quick Sort.

(6+6+6)

7. (a) (i) Prove that if Y ∈ NP-complete. Then Y ∈ P if P = NP.

(ii) Prove that if P = NP then NP = Co-NP.

(b) Define vertex cover problem. Write algorithm for vertex cover problem. Find the vertex cover for the given graph (I).



(c) Illustrate the operation of RADIX- SORT on the array.
A = <62, 234, 456, 750, 3, 10, 789, 98, 65, 5, 21, 983>

(6+7+5)

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