

B3.2-R4: DISCRETE STRUCTURE

NOTE:

1. Answer question 1 and any FOUR from questions 2 to 7.
2. Parts of the same question should be answered together and in the same sequence.

Time: 3 Hours

Total Marks: 100

1.

- a) Let $f : R \rightarrow R$ be a function defined by $f(x) = 1 - x^3$. If $g : R \rightarrow R$ is such that $g(x) = x - 1$, find $g \circ f$ and $f \circ g$. Is $g \circ f$ equals $f \circ g$?
- b) Write the truth table for $(\sim A \rightarrow B) \wedge (\sim B \rightarrow A)$.
- c) A graph has degree sequence 5, 5, 4, 4, 3, 3, 3, 3. How many edges does it have?
- d) Draw the Venn diagram for $(A - B) \cup (B - A)$ and $(A \cup B) - (A \cap B)$.
- e) Let $A = \{1, 2, 3, 4, 5, 6, 7, 8\}$ Write the following permutation as the product of disjoint cycles:

$$\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 4 & 3 & 2 & 5 & 1 & 8 & 7 & 6 \end{pmatrix}$$
- f) Let G be a group, and let a and b be the elements of G . Then prove that the equation $a * x = b$ has a unique solution in G .
- g) Find a deterministic finite accepter that recognizes the set of all strings on $\Sigma = \{a, b\}$ starting with the prefix ab .

(7x4)

2.

- a) Let $A = \{2, 3, 6, 12\}$ and let R and S be the relations on A defined by xRy if '2 divides $(x-y)$ '; and xSy if '3 divides $(x-y)$ '.
Compute $R \cap S$.
- b) Solve the recurrence relation $a_n = 4a_{n-1} - 4a_{n-2}, n \geq 2$ with initial conditions $a_0 = 6, a_1 = 8$.
- c) Find the adjacency matrix of the graph G_1 and G_2 shown in Fig.1. Are these graphs isomorphic? If so, deduce the permutation matrix P and show that $PA_1P^T = A_2$, where A_1 and A_2 are the adjacency matrix of the graphs G_1 and G_2 respectively.

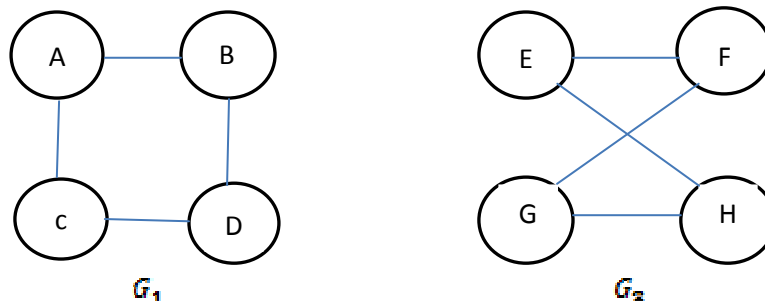


Fig.1

(4+6+8)

3.

- a) What are planer graph? Find whether the following graph is planer or not? How many regions are there? List the edges that form the boundary of each region. Which region is exterior?

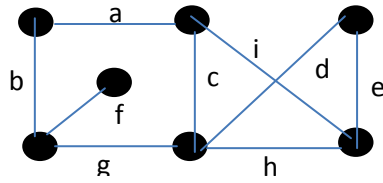


Fig.2

- b) Describe the Prim's algorithm to find a minimum spanning tree in a connected weighted graph with $n > 1$ vertices. Hence use it to find the minimum spanning tree of the following graph (Fig. 3) by starting at the vertex **A**.

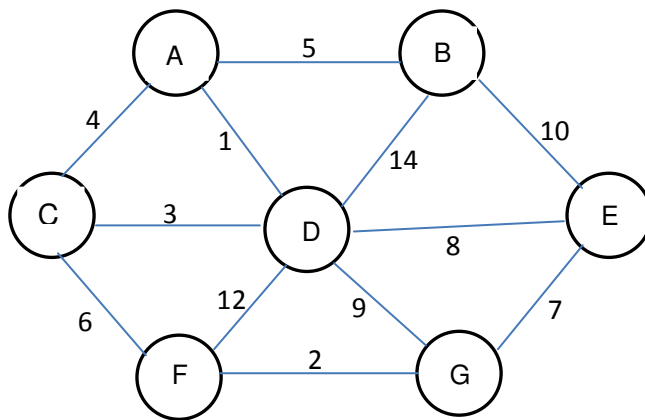


Fig.3

(8+10)

4.

- a) Let $A = \{1, 2, 3, 4, 6, 8, 9, 12, 18, 24\}$. Consider the partial order of divisibility on **A**. Draw the Hasse diagram of the poset (A, \leq) . Is (A, \leq) a lattice? Explain.
- b) Let $\langle f_n \rangle$ be the Fibonacci Sequence. Use mathematical induction to show that

$$\sum_{k=1}^n f_k = f_{n+2} - 1 \text{ for all } n \geq 1.$$

(6+12)

5.

- a) Sort the list 3, 5, 1, 9, 7, 10, 2 into increasing order using the Merge Sort. Explain the steps involved very clearly.
- b) Which are true?

i) $n^2(1 + \sqrt{n}) = O(n^2 \log_2 n)$

ii) $\log_2 n = O(n^{-\frac{1}{2}})$

- c) Draw the state diagram for the NFA (non deterministic finite automata) for which the state table is given in Table: 1 and the accepting states are S_1 and S_3 . Find also the language accepted by this NFA.

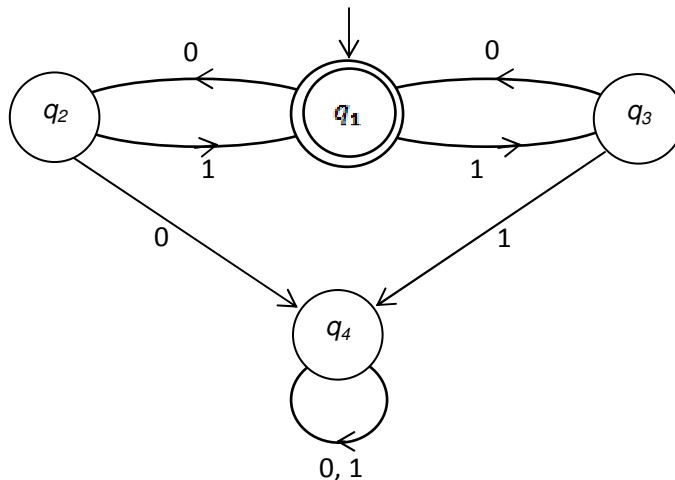
Table: 1

| I → S ↓ | f | |
|------------|-------------|-------------|
| | a | b |
| S_0 | S_2 | S_1 |
| S_1 | S_1, S_2 | S_3 |
| S_2 | \emptyset | \emptyset |
| S_3 | S_2, S_3 | S_2 |

(8+4+6)

6.

- a) In a group of 97 students, the number of students taking Mathematics is twice the number taking Economics. 53 students take exactly one of the subjects, and 15 are taking neither course. How many students are taking Economics? How many as taking Mathematics?
- b) State the generalized pigeonhole principle. Show that among six persons, there are three persons who are mutual friends or there are three persons who are completely stranger to each other.
- c) Find the language accepted by the following automata:



(6+4+8)

7.

- a) Simplify the Boolean function $f(a,b,c,d) = \sum(0,2,6,7,8,9,13,15)$ using Karnaugh map method.
- b) Let A be the set of real numbers except -1, that is, $A = \mathbb{R} - \{-1\}$. Let '*' be an operation on A defined by

$$a * b = a + b + ab$$
 show that
 i) '*' is an binary operation.
 ii) '*' is cumulative and associative
 iii) the identity element is 0.
 Also, find whether (A, *) forms a group or not.

(8+10)