# B4.1-R4: COMPUTER BASED STATISTICAL & NUMERICAL METHODS

**NOTE:**

| | |
|---|---|
| **1.** | **Answer question 1 and any FOUR from questions 2 to 7.** |
| **2.** | **Parts of the same question should be answered together and in the same sequence.** |

**Time: 3 Hours** **Total Marks: 100**

**1.**

a) A bag contains 3 red, 6 white and 7 blue balls. Two balls are drawn at random. Find the probability that (i) both the balls are white (ii) one ball is red and the other is white.

b) If $u = \frac{4x^2 y^3}{z^4}$ and errors in *x*, *y*, *z* be 0.001, compute the relative maximum error in *u* when $x = y = z = 1$.

c) Identify the following statement true or false, justify your answer.

“For any binomial distribution mean is 5 and standard deviation is 3”.

d) Let *X* be a random variable with probability distribution

| | | | | |
|---|---|---|---|---|
| $X$ : | 0 | 1 | 2 | 3 |
| $f(X)$ : | $\frac{1}{3}$ | $\frac{1}{2}$ | 0 | $\frac{1}{6}$ |

Find the expected value of $X^2$ and $(X-1)^2$.

e) Find the missing entry of the following finite difference table:

| | | | | | |
|---|---|---|---|---|---|
| $x$ : | 0 | 1 | 2 | 3 | 4 |
| $y$ : | 1 | 3 | 9 | - | 81 |

f) Suppose that the cumulative function of the random variable X is given by

$$f(x) = 1 - e^{-x^2},\ x > 0$$

Evaluate

i) E(X), and

ii) Var (X), Variance of X.

g) A random variable X is normally distributed with mean 50 and variance 25. Determine P( |X – 50| < 8).

**(7x4)**

**2.**

a) Evaluate $\sqrt{12}$ to four decimal places by Newton-Raphson iterative method.

b) Given that

| | | | | | | |
|---|---|---|---|---|---|---|
| $x$ : | 1 | 2 | 3 | 4 | 5 | 6 |
| $y(x)$ : | 0 | 1 | 8 | 27 | 64 | 125 |

Using Newton interpolation formula find the value of $f(2.5)$.

c) Find the step size h so that the error for the composite trapezoidal rule is less than 5 x $10^{-9}$ for the approximation $\int_2^7 dx \mid x$.

**(6+6+6)**

**3.**

a) Evaluate $\int_0^6 \frac{dx}{1+x^2}$ by Simpson's one-third rule (divide the interval of integration into six equal parts).

b) Solve the following system of linear equations by Guass elimination method

$$x + 3y + 6z = 2$$
$$3x - y + 4z = 9$$
$$x - 4y + 2z = 7$$

c) Consider the following system of linear equations:

5x + 3y = 6
4x – 2y = 8

Can either Gauss-Jacobi or Gauss-Seidel iteration be used to solve this linear system? Why?

**(6+6+6)**

**4.**

a) The probability density function of a random variate *X* is given by

$$p(x) = \begin{cases} k & \text{if } x = 0 \\ 2k & \text{if } x = 1 \\ 3k & \text{if } x = 3 \\ 0 & \text{otherwise} \end{cases}$$

i) Find the value of *k.*

ii) Evaluate $P(X < 2), P(X \le 2), P(0 < X < 2)$.

iii) Find distribution function of *X.*

b) Given below the distribution of digits in numbers chosen at random from a telephone directory:

| digit : | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | Total |
|---|---|---|---|---|---|---|---|---|---|---|---|
| frequency : | 1026 | 1107 | 997 | 966 | 1075 | 933 | 1107 | 972 | 964 | 853 | 10,000 |

Test the result with the hypothesis whether the digits may be taken to occur equally frequently in the directory. (Given $\chi^2_{0.05}$ for 9d.f. = 16.919 and $\chi^2_{0.05}$ for 10d.f. = 18.307)

**(10+8)**

**5.**

a) Use Poisson distribution to find the probability that at most 5 defective fuses will be found in a box of 200 fuses if experience show that 2 percent of such fuses are defective. (Given $e^{-4} = 0.0183$)

b) Two populations have their means equal but standard deviation of one is twice the other. Show that in samples of size 2000 from each drawn under simple sampling condition, the difference of means will, in all probability, not exceed 0.15 times the smaller standard deviation.

c) Using Least Square Method, Obtain the regression of *Y* on *X* from the following data:

| | | | | | |
|---|---|---|---|---|---|
| $X$ : | 1 | 2 | 3 | 4 | 5 |
| $Y$ : | 2 | 4 | 5 | 3 | 6 |

**(6+6+6)**

**6.**

a) Two random variables X and Y have the following joint probability density function:

$$f(x, y) = 2 - x - y; \qquad 0 \le x \le 1,\ 0 \le y \le 1$$
$$= 0, \qquad \text{otherwise}$$

Find the
- i) Marginal probability density function of *X* and *Y*
- ii) Conditional density functions
- iii) Var(*X*) and Var(*Y*).

b) Given the coefficient of correlation *r* = 0.7 and sample size *N* = 64, find the probable error and standard error of *r*. Use them to find
- i) 50% limits for population correlation coefficient.
- ii) 68.26% limits for population correlation coefficient.
- iii) The most probable limits for population correlation coefficient.

**(9+9)**

**7.**

a) For random sampling from normal population $N(\mu, \sigma^2)$, find the maximum likelihood estimators for
- i) $\mu$ when $\sigma^2$ is known
- ii) $\sigma^2$ when $\mu$ is known
- iii) The simultaneous estimation of $\mu$ and $\sigma^2$

b) Two samples from two large populations gave the following results –

| | Mean | S.D. | Sample size |
|---|---|---|---|
| I Sample | 250 | 40 | 400 |
| II Sample | 220 | 55 | 400 |

Test the null hypothesis $H_0 : \mu_1 = \mu_2$ against $H_1 : \mu_1 \ne \mu_2$ at 1% level of significance.

**(9+9)**