B4.1-R4: COMPUTER BASED STATISTICAL & NUMERICAL METHODS

NOTE:

1.	**Answer question 1 and any FOUR from questions 2 to 7.**
2.	**Parts of the same question should be answered together and in the same sequence.**
3.	**Only Non-Programmable and Non-Storage type Scientific Calculator allowed.**

Time: 3 Hours **Total Marks: 100**

1.

a) *A* speaks the truth in 75% cases and *B* in 80% of the cases. In what percentage of cases are they likely to contradict each other in stating the same fact?

b) Find the percentage error in time period $T = 2\pi\sqrt{\frac{l}{g}}$ for l = 1m if error in measurement of l is 0.01.

c) Establish the formula $\sigma^2{}_{x-y} = \sigma^2{}_x + \sigma^2{}_y - 2r\sigma_x\sigma_y$, where each notation has their usual meaning.

d) Six coins are tossed 6400 times. Using Poisson distribution, what is approximate probability of getting six heads *x* times.

e) Find the polynomial which takes the following values

x :	0	1	2	3
f(*x*) :	1	2	1	10

f) Let X be a normal variate representing the adult male height. Suppose the mean and standard deviation of X are 68.6" and 2.74". Find the probability that a randomly chosen adult male is talks then 72". (68.6" = 68.6 inch; 2.74" = 2.74 inch; 72" = 72 inch)

g) Find the second derivative, f"(x) at x=0.3 for the function f(x) given by

x	0.1	0.2	0.3	0.4	0.5	0.6
f(*x*)	0.425	0.475	0.400	0.450	0.525	0.675

(7x4)

2.

a) Use the Newton-Raphson method to find the root of the equation $x^4 - x - 10 = 0$, correct to three places of decimal.

b) Given $f(1) = 4, f(2) = 5, f(7) = 5, f(8) = 4$. Find the value of $f(6)$, using Lagrange's interpolation formula.

c) The random variable *x* has the p.d.f. given by

$$f(x) = xe^{-\frac{x^2}{2}}; \qquad x \geq 0$$
$$= 0, \qquad \text{otherwise}$$

Find the mean and variance of *x*.

(6+6+6)

3.

a) Evaluate $\int_0^1 \frac{dx}{1+x^2}$ using

i) Simpson's one-third rule taking $h = \frac{1}{4}$

ii) Simpson's Three-eight rule taking $h = \frac{1}{6}$. Hence compute an approximate value of π.

b) Using Gauss-Seidel method, solve the following system of linear equations

$$27x + 6y - z = 85$$
$$6x + 15y + 2z = 72$$
$$x + y + 54z = 110$$

Take the initial value as (0, 0, 0).

(9+9)

4.

a) The joint probability distribution of the bivariate random variable (X, Y) is

$$f(x, y) = \begin{cases} x + y & : \quad 0 < x < 1,\ 0 < y < 1 \\ 0 & : \quad \text{elsewhere} \end{cases}$$

Compute $E(X)$ and $E(Y)$. Also, find the correlation coefficient of *X* and *Y*.

b) A coin was tossed 400 times and the head turned up 216 times. Test the hypothesis that the coin is unbiased.

(9+9)

5.

a) A manufacturer of cotter pins knows that 5 percent of his product is defective. If he sells pins in boxes of 100 and guarantees that no more than 10 pins will be defective what is the approximate probability that a box will fail to meet the guaranted quality.

b) In a continuous distribution whose relative frequency density is given by $f(x) = \frac{3}{4}x(2 - x)$, the variable ranges from 0 to 2. Show that the distribution is symmetrical with mean $x = 1$ and variance $\frac{1}{5}$. Show that the third moment about

$x = 0$ is $\frac{1}{5}$. Verify $\mu_3 = 0$.

c) The following table gives age (*x*) in years of cars and annual maintenance cost (*y*) in hundred rupees:

x :	1	3	5	7	9
y :	15	18	21	23	22

Estimate the maintenance cost for a 4 year old car after finding the regression equation.

(6+6+6)

6.

a) If $x_1, x_2, x_3, \ldots\ldots\ldots\ldots x_n$ is a random sample from a normal distribution $N(\mu, 1)$ show that $t = \frac{1}{N}\sum_{i=1}^{n} x_i^2$ is an unbiased estimator of $\mu^2 + 1$.

b) A set of five similar coins is tossed together 320 times and the result is

x (no. of heads)	0	1	2	3	4	5
f (frequency of tosses)	6	27	72	112	71	32

Test the hypothesis that the data follow a binomial distribution.

(9+9)

7.

a) The total of 2000 students appeared in an examination. Distribution of marks is assumed to be normal with mean $\mu = 30$ and $\sigma = 6.25$. How many students are expected to get marks between 20 and 40 or $P(20 < X < 40)$.

b) For random variables X and Y, the joint probability density function is given by

$$f(x, y) = 2; \qquad 0 < x < 1,\ 0 < y < x$$
$$= 0, \qquad \text{elsewhere}$$

Find the marginal density function of *X* and *Y*. Also obtain conditional density functions.

(9+9)