C4-R4: ADVANCED ALGORITHMS

NOTE:

1. **Answer question 1 and any FOUR from questions 2 to 7.**
2. **Parts of the same question should be answered together and in the same sequence.**

Time: 3 Hours **Total Marks: 100**

1.

a) Explain why the statement "The running time of algorithm A is at least $O(n^2)$" is meaningless.
b) What is the time efficiency of the DFS-based algorithm for topological sorting?
c) What are the features of an algorithm?
d) How the greedy paradigm of an Algorithm differs from that of Dynamic Programming?
e) A sequence of n operations is performed on a data structure. The ith operation costs 'i' if 'i' is an exact power of 2, and 1 otherwise. Use aggregate analysis to determine the amortized cost per operation.
f) Prove that the time efficiency of Warshall's algorithm is cubic.
g) When can a search path be terminated in a branch-and-bound algorithm?

(7x4)

2.

a) Solve the following recurrence equations
   1. $T(n)=3T(n/3 + 5) + n/2$
   2. $T(n)= T(n-2) + 2\log n$
b) What is the largest number of key comparisons made by binary search in searching for a key in the array: 3 14 27 31 39 42 55 70 74 81 85 93 98? List all the keys of this array that will require the largest number of key comparisons when searched by binary search method.
c) List the functions below from lowest order to highest order. If any two or more are of the same asymptotic order, group them together.

   $\log \log n$, 2^n, $(3/2)^n$, n^3, e^n, $\log_2 n$, $n \log n$, 20, $(n^2-1)/(n-1)$, 100, $n\, 4^{\log n}$, $\log \log n^2$

(6+6+6)

3.

a) Write an Algorithm to solve Knapsack Problem using Greedy. Using this greedy algorithm find an optimal solution for knapsack instance n=7, M = 15, $(P_1, P_2, P_3, P_4, P_5, P_6, P_7) = (10, 5, 15, 7, 6, 18, 3)$ and $(W_1, W_2, W_3, W_4, W_5, W_6, W_7) = (2, 3, 5, 7, 1, 4, 1)$.
b) Using Dynamic Programming algorithm determine an Longest Common Sequence of (A, T, G, T, T, A, T) and (A, T, C, G, T, A, C).
c) Give the implementation of Tower of Hanoi problem using Recursion.

(10+6+2)

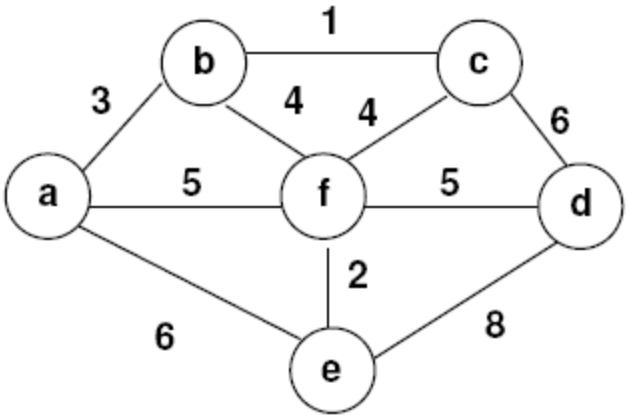
4.

a) Explain the use of Backtracking method for solving n- Queen Problem along with algorithm.
b) What are commonalities and differences between backtracking and branch and bound algorithms?
c) Give an example of an algorithm that should not be considered an application of the brute-force approach.

(9+6+3)

5.

a) Write the Prim's algorithm for finding the minimum spanning tree. Apply the algorithm on following graph:



b) Use Strassen's matrix multiplication algorithm to multiply

$$X = \begin{bmatrix} 3 & 2 \\ 4 & 8 \end{bmatrix} \text{ and } Y = \begin{bmatrix} 1 & 5 \\ 9 & 6 \end{bmatrix}$$

(10+8)

6.

a) How many character comparisons will the Boyer-Moore algorithm make in searching for each of the following patterns in the binary text of 1000 zeros?

i) 00001
ii) 10000
iii) 01010

b) Define NP, NP-Complete and NP-Hard problem with example.

(9+9)

7.

a) Write an algorithm for merge sort and analyze worst case complexity.
b) Discuss travelling salesman problem.
c) Euclid's algorithm is known to be in O(log n). If it is the algorithm that is used for computing gcd(m, n), what is the efficiency of the algorithm for computing lcm(m, n)?

(10+6+2)