B3.2-R4: DISCRETE STRUCTURE

NOTE:

1.	**Answer question 1 and any FOUR from questions 2 to 7.**
2.	**Parts of the same question should be answered together and in the same sequence.**

Time: 3 Hours **Total Marks: 100**

1.

a) In a class of 25 students, 13 have taken mathematics, 9 have taken mathematics but not biology. Find the number of students who have taken mathematics and biology and those who have taken biology but not mathematics.

b) Using principle of mathematical induction, prove that proposition (for $n \geq 0$):

$P(n) = 2^0 + 2^1 + 2^2 + 2^3 + 2^4 + \ldots\ldots + 2^n = 2^{n+1} - 1.$

c) Prove that $(A \times B) \cap (A \times C) = A \times (B \cap C)$.

d) Find P(B/A) if (i) A is a subset of B (ii) A and B are mutually exclusive.

e) Without using truth table, prove the logical equivalence of

$[d \rightarrow ((-a) \wedge b) \wedge c]$ and $\neg[(a \vee (\neg(b \wedge c))) \wedge d]$.

f) Draw the Hasse diagram of divisors of 70 and show that the set of all divisors of 70 form a lattice.

g) Show that the argument $p \rightarrow q, \neg p \mid\!\rightarrow \neg q$ is a fallacy.

(7×4)

2.

a) Solve the recurrence relation $t_n = 5t_{n-1} - 8t_{n-2} + 4t_{n-3}$ for $n \geq 3$,

subject to initial conditions $t_n = n$ if $n = 0$, 1, and 2.

b) Given A= {1, 2, 3, 4} and B= {x , y, z}. Let R be the following relation from A to B,
R = {(1 , y), (1, z), (3, y), (4, x), (4, z)}.
- i) Determine the matrix of the relation R.
- ii) Draw the arrow diagram of R.
- iii) Find the inverse relation R^{-1} of R.
- iv) Determine the domain and range of R.

(10+[4x2])

3.

a) Show that the set of all positive rational numbers forms an abelian group under the composition * defined by $a * b = \dfrac{ab}{4} \;\forall\, a, b \in \mathbb{Q}^+$.

b) Prove the relation R in the set N of natural numbers defined by $a\,R\,b \Leftrightarrow (a - b)$ is divisible by *n* where n ϵ *N* is an equivalence relation.

(9+9)

4.

a) An urn contains 9 red, 7 white and 4 black balls. A ball is drawn at random. What is the probability that the ball drawn will be
- i) Red
- ii) White
- iii) White or Black
- iv) Red or Black

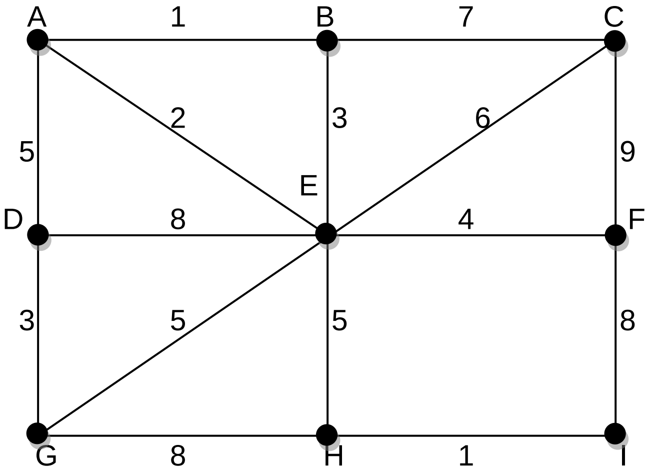
b) Show that the maximum number of edges in a simple graphs with *n* vertices is $\frac{n(n-1)}{2}$.

c) Can a simple graph exist with 15 vertices, each of degree 5? Justify.

(8+5+5)

5.

a) Find the minimum spanning tree (MST) for the following weighted graph using Kruskal's algorithm. Also explain the steps involved.



b) i) Draw a graph with six vertices which is Hamiltonian but not Eulerian.

ii) Draw a graph with six vertices which is Eulerian but not Hamiltonian.

(10+[4+4])

6.

a) Let M be the automaton as follows, where A is the input set, S is the state set, and Y is the accepting (yes) state: A= {a, b}, S=[s_o, s_1 s_2}, Y= {s_1}.
Suppose s_o is the initial state of M and next state function F of M is given in the following table.

F	a	b
s_0	s_0	s_1
s_1	s_1	s_2
s_2	s_2	s_2

i) Draw the state diagram D (M) of the automaton M.

ii) Describe the language L (M) accepted by the automaton M.

b) Let A = {0, 1}, construct a finite state automaton M such that L (M) will consists of

i) those words in which number of 0's and 1's are even.

ii) those words in which number of 1's are odd.

([5+4]+[5+4])

7.

a) Let a and b be the positive integers, and Q is defined recursively as follows:

$$Q(a, b) = \begin{cases} 0 & \text{if } a < b, \\ Q(a-b, b)+1, & \text{if } b \leq a. \end{cases}$$

i) Find Q (2, 5).

ii) Find Q (12, 5)

iii) What does this function Q do? Also find Q (5861, 7).

b) Let $E = xy' + xyz' + x'yz'$ is a Boolean expression, then prove that:

i) $xz' + E = E$ ii) $x + E \neq E$ iii) $z' + E \neq E$

c) Suppose $P(n) = a_0 + a_1 n + a_2 n^2 + \cdots .. a_m n^m$ has degree m. Prove that $P(n) = O(n^m)$.

([1+3+2]+[3x2]+6)