B4.1-R4: COMPUTER BASED STATISTICAL & NUMERICAL METHODS

NOTE:

1. **Answer question 1 and any FOUR from questions 2 to 7.**
2. **Parts of the same question should be answered together and in the same sequence.**
3. **Only Non-Programmable and Non-Storage type Scientific Calculator allowed.**

Time: 3 Hours **Total Marks: 100**

1.

a) Three distinct integers are chosen at random from the first 20 positive integers. Compute the probability that their product is even.

b) Find the probability density function of the random variable X whose cumulative distribution function is

$$F(x) = \begin{cases} 0.00 \text{ if } x < -1 \\ 0.25 \text{ if } -1 \le x < 1 \\ 0.50 \text{ if } 1 \le x < 3 \\ 0.75 \text{ if } 3 \le x < 5 \\ 1.00 \text{ if } x \ge 5\ . \end{cases}$$

c) If the probability density functions of the random variable X is

$f(x) = (1 - p)^{x-1}\, p$ if x = 1, 2, 3, 4, ...

= 0 otherwise,

then what is the expected value of X?

d) Numerically approximate the integral $\int_0^2 \left(2 + \mathrm{Cos}\left[2\sqrt{x}\,\right]\right) dx$ by using Simpson's rule with the number of subintervals, n = 6.

e) Let $X_1, X_2, ..., X_{10}$ be independent Poisson random variables with mean 1. Use the central limit theorem to approximate $P(X_1 + X_2 + ... + X_{10} \geq 15)$.

f) A manufacturer of electric bulbs claims that the percentage of defectives in his product doesn't exceed 6. A sample of 40 bulbs is found to contain 5 defectives. Would you consider the claim justified at 5% level of significance.

g) Let $p(x_1, x_2) = \frac{1}{16}, x_1, x_2 = 1, 2, 3, 4,$ zero elsewhere be the joint probability mass function (pmf) of X_1 and X_2. Are X_1 and X_2 independent?

(7x4)

2.

a) In a school of 1200 students, 250 are seniors, 150 students take math, and 40 students are seniors and are also taking math. What is the probability that a randomly chosen student, who is a senior, is taking math?

b) Suppose we toss a pair of fair, four-sided dice, in which one of the dice is RED and the other is BLACK. We'll let:

X = the outcome on the RED die = {1, 2, 3, or 4}

Y = the outcome on the BLACK die = {1, 2, 3, or 4}

What is the mean of X? And, what is the mean of Y?

c) Let $f_{1/2}\,(x_1/x_2) = c_1\, x_1 / x_2^2$, $0 < x_1 < x_2$, $0 < x_2 < 1$, zero elsewhere, and $f_2(x_2) = c_2\, x_2^4$, $0 < x_2 < 1$, zero elsewhere, denote, respectively , the conditional pdf of X_1, given $X_2 = x_2$, and the marginal pdf of X_2. Determine c_1 and c_2 and hence, the joint pdf of X_1 and X_2.

(6+6+6)

3.

a) Use Gaussian elimination to solve the system of linear equations

$$2x_2 + x_3 = -8$$
$$x_1 - 2x_2 - 3x_3 = 0$$
$$-x1 + x_2 + 2x_3 = 3$$

b) Compute f(0.3) for the data

x	0	1	3	4	7
f	1	3	49	129	813

using Lagrange's interpolation formula.

c) Find the real root of the equation $x^3 + x^2 = 1$ by iteration method.

(6+6+6)

4.

a) Show that for the geometric distribution with parameter $\boldsymbol{\theta}$**,** the mean, $\mu = \frac{1}{\theta}$ and variance, $\sigma^2 = \frac{1-\theta}{\theta^2}$.

b) Show that for normal distribution median and mode coincide.

c) Show that if a random variable has a uniform density with the parameters α and β, the *r*th moment about the mean equals

$$\mu_r = \begin{cases} 0, & \textit{when r is odd} \\ \frac{1}{r+1}\left(\frac{\beta-\alpha}{2}\right)^r, & \textit{when r is even.} \end{cases}$$

(6+6+6)

5.

a) Let X and Y have the joint pdf

f(x, y) = 2 if x > 0, y > 0, x + y < 1

=0 elsewhere.

Find Cov(X, Y). Are X and Y independent?

b) A biased coin, which lands heads with probability 1/10 each time it is flipped, is flipped 200 times consecutively. Give an upper bound on the probability that it lands heads at least 120 times.

c) Given the joint density

$$f(x,y) = \begin{cases} 6x, & 0 < x < y < 1 \\ 0, & \text{elsewhere.} \end{cases}$$

Find the regression equation of X on Y.

(5+4+9)

6.

a) Diets A and B were administered simultaneously to two sets of children. A random sample of 12 children from the set (to which Diet A was administered) and a random sample of 15 children from the set (to which Diet B was administered) were chosen and the gains in weight after a certain period of time were collected as given below:

Diet	**Gain in Weight (in Grams)**														
Diet A	25	32	30	34	24	14	32	24	30	31	35	25			
Diet B	44	34	22	10	47	31	40	30	32	35	18	21	35	29	22

Test at 5% level of significance, if the two diets differ significantly as regards to their effect on increase of weight.

b) A sample of 50 units was considered for intensive inspection, and the units were grouped as having 0, 1, 2, 3, or 4 defectives. The frequency table of observations on the random variable X for number of defective units is given as:

X	0	1	2	3	4
Frequency	4	21	10	13	2

Test at 1% level of significance whether this data can be considered to follow a binomial distribution.

(9+9)

7.

a) Two samples of 10 and 15 leas have been taken and weighed. The sample averages are 9.8 g and 9.2 g. The sample standard deviations are 0.3 g and 0.4 g, respectively, and the population variances are not known and not equal. Test at 1% the two samples could be from the same population?

b) Let the sample values 2, 3, 5, 6, 8, 9, 11 and 12 are drawn from a uniform population, and let the unknown parameter of minimum value be 'a' and the parameter of maximum value be 'b' in the population. Estimate the range (a, b) of this population using the method of moments.

c) If X_1, X_2, ..., X_n is a random sample from a distribution with density function

$$f(x; \theta) = 1/\theta \quad \text{if } 0 < x < \theta$$
$$= 0 \text{ otherwise,}$$

then what is the maximum likelihood estimator of θ?

(6+6+6)

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