

C0-R4.B1: ELEMENTS OF MATHEMATICAL SCIENCES

NOTE:

1. Answer question 1 and any FOUR from questions 2 to 7.
2. Parts of the same question should be answered together and in the same sequence.

Time: 3 Hours

Total Marks: 100

1.

- a) Find the values of λ for which the vectors $(1, -2, \lambda)$, $(2, -1, 5)$ and $(3, -5, 7\lambda)$ are linearly dependent.
- b) Evaluate $\lim_{x \rightarrow 0} \frac{x \cos x - \text{Log}_e(1+x)}{x^2}$.
- c) Evaluate $\int_0^{\pi/2} \frac{\sqrt{\sin x}}{\sqrt{\sin x} + \sqrt{\cos x}} dx$.
- d) Examine the character of the series $\sum_{n=2}^{\infty} \frac{(-1)^{n-1} x^n}{n(n-1)}$, $0 < x < 1$.
- e) A bag contains 7 white, 6 red and 5 black balls. Two balls are drawn at random. Find the probability that they will both be white.
- f) If $y = \tan^{-1}\left(\frac{\sin x}{1 + \cos x}\right) + \cos^{-1}(\sin x)$ then find $\frac{dy}{dx}$.
- g) Find the equation of the ellipse, whose length of the major axis is 20 and foci are $(0, \pm 5)$.

(7x4)

2.

- a) Using Gauss-Jordan method, find the inverse of the following matrix: $A = \begin{bmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{bmatrix}$.
- b) For what values of k , the equation $x + y + z = 1$, $2x + y + 4z = k$, $4x + y + 10z = k^2$, have a solution and solve them completely in each case.

- c) Find the Eigen values and the eigenvectors of the following matrix: $A = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$.

(6+6+6)

3.

- a) Find the asymptotes of the curve $x^3 + 3x^2y - 4y^3 - x + y + 3 = 0$.
- b) Find the absolute maximum and minimum values of a function f given by:
 $f(x) = 12x^{4/3} - 6x^{1/3}; x \in [-1, 1]$.
- c) Integrate the following function $f(x)$ w.r.t. x : $f(x) = \frac{\tan^4 \sqrt{x} \sec^2 \sqrt{x}}{\sqrt{x}}$.

(6+6+6)

4.

- a) Discuss the convergence of the series $\sum_{n=1}^{\infty} \frac{4.7.....(3n+1)}{1.2.3.....n} x^n$.
- b) Expand $e^{\sin x}$ by Maclaurin's series up to the terms containing x^4 .
- c) Expand $f(x) = \log_e x$ in ascending powers of $(x-1)$ by Taylor's theorem and hence find the value of $\log_e(1.1)$.

(6+6+6)

5.

- a) Find the equation of the hyperbola where foci are $(0, \pm 12)$ and the length of the latus rectum is 36.
- b) If α and β are the roots of the quadratic equation $3x^2 - 4x + 1 = 0$, then find the quadratic equation whose roots are α^2 / β and β^2 / α .
- c) Find the angle between the lines $y - \sqrt{3}x - 5 = 0$ and $\sqrt{3}y - x + 6 = 0$.

(6+6+6)

6.

- a) Let X be a random variable with the following probability distribution:

$X :$	-3	6	9
$P(X = x) :$	1/6	1/2	1/3

Find $E(X)$, $E(X^2)$ and using the laws of expectation, evaluate $E(2X + 1)^2$.

- b) The probability that a pen manufactured by a company will be defective is $1/10$. If 12 such pens are manufactured, find the probability that
- i) exactly two will be defective, ii) at least two will be defective, iii) none will be defective.
- c) If the probability of a bad reaction from a certain injection is 0.001, then determine the chance that out of 2000 individuals more than two will get a bad reaction.

(6+6+6)

7.

- a) In experiment on pea breeding, the following frequencies of seed were obtained:

Round & Yellow	Wrinkled & Yellow	Round & Green	Wrinkled & Green	Total
315	101	108	32	556

Theory predicts that frequencies should be in proportions 9:3:3:1. Examine the correspondence between theory and experiment.

- b) Find the moment generating function of the exponential distribution $f(x) = (1/c)e^{-x/c}$; $0 \leq x < \infty$; $c > 0$. Hence find its mean and Standard Deviation.

(9+9)