C4-R4: ADVANCED ALGORITHMS

NOTE:

1. **Answer question 1 and any FOUR from questions 2 to 7.**
2. **Parts of the same question should be answered together and in the same sequence.**

Time: 3 Hours **Total Marks: 100**

1.

a) Solve the recurrence: $T(n) = 5T(n/5) + \sqrt{n}$; $T(1) = 1$; $T(0) = 0$.

b) What are the similarities and dissimilarities between 'Divide and Conquer method' and 'Dynamic Programming'?

c) Discuss the disadvantage of Naïve String Matching with an example.

d) For each of the following functions, indicate how much the function's value will changes if its argument is increased fourfold .i) n^3 ii) 2^n

e) How can we use Prim's algorithm to find a spanning tree of a connected graph with no weights on its edges? Is it a good algorithm for this problem?

f) Give two examples of problems which are
   i) NP but not NP-Complete
   ii) NP Complete, and
   iii) NP-Hard but not NP-Complete

g) Which sorting algorithm is the best: Quick sort, Merge sort or Heap sort and why?

(7x4)

2.

a) Working modulo q = 11, how many spurious hits does the Rabin-Karp matcher encounter in the text T = 3141592653589793 when looking for the pattern P = 26?

b) i) What is min-heap? Is an array that is in sorted order a min-heap?
   ii) What is the effect of calling MIN-HEAPIFY (A, i) for i>size[A]/2?

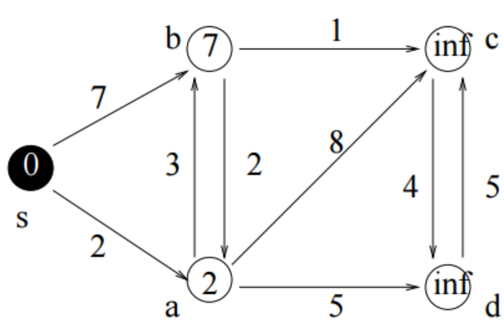
c) Illustrate the operation of RADIX-SORT on the following list of English words: COW, DOG, SEA, RUG, ROW, MOB, BOX, TAB, BAR, EAR, TAR, DIG,BIG, TEA, NOW, FOX.

(6+6+6)

3.

a) A sequence of n operations is performed on a data structure. The ith operation costs C_i if i is an exact power of 2, and 1 otherwise. Use the accounting method to determine the amortized cost per operation.

b) Find the shortest path using Djikstra's algorithm for the following weighted digraph:



c) Consider the following algorithm.
Algorithm Secret(A[0..n − 1]) //Input: An array A[0..n − 1] of n real numbers
minval ← A[0]; maxval ← A[0]
for i ← 1 to n − 1 do
if A[i] < minval
minval ← A[i]
if A[i] > maxval

maxval ← A[i]
return maxval – minval

i) What does this algorithm compute?
ii) What is its basic operation?
iii) How many times is the basic operation executed?
iv) What is the efficiency class of this algorithm?
v) Suggest an improvement or a better algorithm altogether and indicate its efficiency class. If you cannot do it, try to prove that, in fact, it cannot be done.

(6+6+6)

4.

a) For each of the following functions, indicate the class Θ(g(n)) the function belongs to. (Use the simplest g(n) possible in your answers.) Prove your assertions.
   i) sqrt(10n^2 + 7n + 3)
   ii) 2n lg(n + 2)^2 + (n + 2)^2 lg n/2
   iii) 2^(n+1) +3^(n−1)

b) You are traveling by a canoe down a river and there are n trading posts along the way. Before starting your journey, you are given for each 1 <= i<=n the fee f[i;j] for renting a canoe from post i to post j. These fees are arbitrary. For example it is possible that f[1;3] = 10 and f[1;4]= 5. You begin at trading post 1 and must end at trading post n (using rented canoes). Your goal is to minimize the rental cost. Give the most efficient algorithm you can for this problem. Be sure to prove that your algorithm yields an optimal solution and analyze the time complexity.

(9+9)

5.

a) Consider an RSA key set with $p = 11$, $q = 29$, $n = 319$, and $e = 3$. What value of d should be used in the secret key? What is the encryption of the message $M = 100$?

b) Consider two strings A = "qpqrr" and B = "pqprqrp". Let x be the length of the longest common subsequence (not necessarily contiguous) between A and B and let y be the number of such longest common subsequences between A and B. Then find x + 10y.

(9+9)

6.

a) A simple path in a graph is a path that does not hit any vertex more than once. The Longest Path problem is as follows:
Input: An undirected graph G and a positive integer K.
Question: Is there a simple path in G that has length at least K?
Show that the Hamilton Path problem can be transformed in polynomial time to the Longest Path problem.

b) Consider two teams, A and B, playing a series of games until one of the teams wins n games. Assume that the probability of *A* winning a game is the same for each game and equal to *p*, and the probability of *A* losing a game is $q = 1 – p$. (Hence, there are no ties.). Let *P(i, j)* be the probability of *A* winning the series if *A* needs *i* more games to win the series and *B* needs *j* more games to win the series.
   i) Set up a recurrence relation for *P(i,j)* that can be used by a dynamic programming algorithm.
   ii) Find probability of *A* winning a 7-game series if the probability of it winning a game is 0.4.
   iii) Write pseudo code of the DP algorithm and determine its time & space efficiencies.

(9+9)

7.

a) Apply counting sort algorithm to sort the list 60, 35, 81, 98, 14, 47. Is this algorithm stable? Is it in place?

b) i) Prove the equality gcd(m, n) = gcd(n, m mod n) for every pair of positive integers m and n.

ii) What does Euclid's algorithm do for a pair of numbers in which the first number is smaller than the second one? What is the largest number of times this can happen during the algorithm's execution on such an input?

(9+9)