1st Semester Diploma Engineering Exam., 2016

ENGINEERING MATHEMATICS—I

Paper : ENM–104

Full Marks : 70

Time : 3 hours

The figures in the margin indicate full marks for the questions

FIRST HALF

Answer Question No. **1** and *any* **three** from the rest

1. Answer any *four* from the following :　　2×4=8

   (a) If $\tan 25° = a$, then find the value of $\tan 155°$ and $\tan 115°$.

   (b) If $\vec{a}$ and $\vec{b}$ are two vectors such that $|\vec{a}| = 3$, $|\vec{b}| = 2$ and $\vec{a} \cdot \vec{b} = 6$, then find the angle between $\vec{a}$ and $\vec{b}$.

   (c) Find the area of the triangle formed by the points $(-2, -2)$, $(2, 2)$ and $(-2\sqrt{3}, 2\sqrt{3})$.

　　　　　　　　　　　　　(Turn Over)

(d) Prove that $\tan 70° = 2\tan 50° + \tan 20°$.

(e) Find the square root of the complex number $1+i$.

(f) Find the 4th-term in the expansion of $\left(x+\dfrac{2}{x}\right)^5$.

2. (a) Prove that

$$\tan^{-1}\frac{1}{4}+2\tan^{-1}\frac{1}{5}=\tan^{-1}\frac{32}{43}$$

(b) Find the coefficient of a^{15} in the expansion of $\left(a^3+\dfrac{2}{a^2}\right)^{10}$.   5+4=9

3. (a) Solve $2\sin^2\theta+3\cos\theta=0$, $0<\theta<360°$.

(b) Find the sum of all integers between 50 and 500, which are divisible by 7.   4+5=9

4. (a) Find the equation of a circle passing through the points (5, 7), (6, 6) and (2, –2). Find its centre and radius.

(b) If $z=x+iy$ and $|2z+1|=|z-2i|$, then prove that $3(x^2+y^2)+4(x+y)=3$.   5+4=9

5. (a) Prove that

$$\cos 20°\cos 40°\cos 60°\cos 80°=\frac{1}{16}$$

(b) If the roots of the equation $ax^2+bx+c=0$ are in the ratio $3:4$, then prove that $49\,ac=12b^2$.   9

6. (a) If $\sin\theta=k\sin(\theta+\phi)$, then show that

$$\tan(\theta+\phi)=\frac{\sin\phi}{\cos\phi-k}$$

(b) Find the lengths of major and minor axes, the coordinates of foci and vertices and the eccentricity of the ellipse, $16x^2+25y^2=400$.   4+5=9

SECOND HALF

Answer Question No. **7** and *any* **three** from the rest

7. Answer any *four* from the following :   2×4=8

(a) Find the domain of definition of the function $f(x)=\sqrt{x^2+x-12}$.

(b) Evaluate :

$$\underset{x\to 0}{\text{Lt}}\ \frac{1-\cos 4x}{x^2}$$

(c) Find $\dfrac{dy}{dx}$, where $y = (\sin x)^{\log x}$.

(d) Find the second-order derivative of the function w.r.t. x, $y = \sin 3x \cos 2x$.

(e) Evaluate :

$$\int x \sin x \, dx$$

(f) Find $\dfrac{dy}{dx}$, if

$$y = \sin^{-1}\left(\dfrac{2x}{1+x^2}\right)$$

8. (a) If $xy = \sin(x + y)$, then prove that

$$\dfrac{dy}{dx} = \dfrac{\cos(x+y) - y}{x - \cos(x+y)}$$

(b) Evaluate :

$$\int \dfrac{x \, dx}{(x^2+1)(x^2+4)}$$

$4+5=9$

9. (a) Evaluate :

(i) $\underset{x \to 0}{\text{Lt}} \dfrac{e^{\sin x} - 1}{\log(1+3x)}$

(ii) $\underset{x \to 0}{\text{Lt}} \dfrac{2^x - 3^x}{x}$

(b) Evaluate :

$$\int \dfrac{dx}{x^2 + 5x + 6}$$

$(3+3)+3=9$

10. (a) If $y = (\tan^{-1} x)^2$, then prove that

$$(1+x^2)^2 y_2 + 2x(1+x^2)y_1 = 2$$

(b) Evaluate :

$$\int \dfrac{x e^x}{(1+x)^2} \, dx$$

$5+4=9$

11. (a) Find the derivative of $\sin^{-1} \dfrac{2x}{1+x^2}$ with respect to $\tan^{-1} \dfrac{2x}{1-x^2}$.

(b) Evaluate :

$$\int_0^{\pi/2} \dfrac{dx}{4 + 5\sin x}$$

$4+5=9$

12. (a) Given

$$\phi(x) = \dfrac{1 - \cos x}{x^2}, \quad \text{when} \quad x \neq 0$$

$$= \dfrac{1}{2}, \quad \text{when} \quad x = 0$$

prove that $\phi(x)$ is continuous at $x = 0$.

(b) Find $\dfrac{d^2y}{dx^2}$, when $x = a(\theta + \sin \theta)$, $y = a(1 - \cos\theta)$ at $\theta = \dfrac{\pi}{2}$.

$4+5=9$

★★★