B3.2-R3: BASIC MATHEMATICS

NOTE:

1.	**Answer question 1 and any FOUR from questions 2 to 7.**
2.	**Parts of the same question should be answered together and in the same sequence.**

Time: 3 Hours **Total Marks: 100**

1.

a) Find the value of x for which the determinant of the matrix $A = \begin{bmatrix} 1 & -2 & 3 \\ 1 & 2 & 1 \\ x & 2 & -3 \end{bmatrix}$ is zero.

b) Let $\frac{1}{\text{Cos}\theta - i\text{Sin}\theta} = A + iB, \quad i = \sqrt{-1}$. Find A and B.

c) A particle is moving in a straight line according to $S=2t^3 - 9t^2 + 24t$, where S is in meters and t is in seconds. Determine the distance and velocity at the end of 2 seconds.

d) Evaluate $\int \frac{\text{Sin}\, x}{1+\text{Cos}^2 x} dx$.

e) Find the centre and radius of the circle $2x^2 + 2y^2 - 12x + 8y - 95 = 0$.

f) Is the series $1^2 + 2^2x + 3^2x^2 + 4^2x^3 + \ldots$
convergent or divergent?

g) Two vectors $\vec{C}$ and $\vec{D}$

$$\vec{C} = 2\hat{i} + 5\hat{j} + 8\hat{k}$$

$$\vec{D} = \hat{i} + p\hat{j} + \hat{k}$$

are perpendicular to each other. Find p.

(7x4)

2.

a) Find out for what values of λ, the equations

$$x + y + z = 1$$
$$x + 2y + 4z = \lambda$$
$$x + 4y + 10z = \lambda^2$$

have a solution.

b) Evaluate the determinant

$$\Delta = \begin{vmatrix} 1 & \text{Sin}\theta & 1 \\ -\text{Sin}\theta & 1 & \text{Sin}\theta \\ -1 & -\text{Sin}\theta & 1 \end{vmatrix}$$

Also prove that $2 \le \Delta \le 4$.

c) Find the characteristic roots and characteristic vectors of a matrix A, where

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 2 & 3 \\ 0 & 0 & 2 \end{bmatrix}$$

(6+6+6)

3.

a) Sketch the graph of the function $y = 4 - x^2$, $0 \le x \le 2$. Determine the area enclosed by the curve, the x-axis and the lines x = -2 and x = 2.

b) The length x of the rectangle is decreasing at the rate of 2cm/sec and the width y is increasing at the rate of 2cm/sec. Find the rate of change of (i) perimeter, (ii) the area of the rectangle when x = 12cm and y = 5cm.

c) Determine $\lim\limits_{x \to 0} \dfrac{\text{Sin}\, 3x}{\text{Sin}\, x}$.

(6+6+6)

4.

a) Evaluate $\int_1^3 \dfrac{(x^2 + 5x + 3)}{(x^2 + 3x + 2)}\, dx$.

b) Evaluate $\int e^x (x^{-1} - x^{-2})\; dx$.

c) Test for positive values of x the convergence of the series $\dfrac{x}{1.2} + \dfrac{x^2}{3.4} + \dfrac{x^3}{5.6} + \dfrac{x^4}{7.8} + \ldots$

(6+6+6)

5.

a) Sketch the curve $x^2 - 4x + (y - 3)^2 = 0$. Also determine the area enclosed by the curve.

b) Express the following matrix A as a sum of symmetric and skew-symmetric matrix

$$A = \begin{bmatrix} -1 & 7 & 1 \\ 2 & 3 & 4 \\ 5 & 0 & 5 \end{bmatrix}.$$

c) Find the middle term in the expansion of $(5 + 2x)^{17}$.

(6+6+6)

6.

a) Obtain the asymptotes parallel to the x-axis of the curve $x^2y - 3x^2 - 5xy + 6y + 2 = 0$.

b) Find the equation of the tangent at $\theta = \dfrac{\pi}{2}$, to the curve

$$x = a(\theta + \text{Sin}\theta)$$
$$y = a(1 + \text{Cos}\theta)$$

c) Obtain k if

$y = a\,\text{Cos}(\log x) + b\,\text{Sin}(\log x)$

and

$ky = (x^2 \dfrac{d^2y}{dx^2} + x \dfrac{dy}{dx})$

(6+6+6)

7.

a) Find the first three terms of Taylor's series for ln(x) about x=2.

b) Find the equation of the ellipse whose foci are (2, 3), (-2, 3) with semi-minor axis $\sqrt{5}$.

c) If $x^3 + [f(x)]^3 - 3ax[f(x)] = 0$, find $f'(x)$ given $ax - [f(x)]^2 \ne 0$.

(6+6+6)