B3.2-R4: DISCRETE STRUCTURES

NOTE:

1.	**Answer question 1 and any FOUR from questions 2 to 7.**
2.	**Parts of the same question should be answered together and in the same sequence.**

Time: 3 Hours **Total Marks: 100**

1.

a) In a class of 120 students, 80 students study Maths, 45 students study History and 20 students neither study History nor study Maths. What is the number of students who study both Maths and History?

b) If the functions f: $R \rightarrow R$ and g : $R \rightarrow R$ are defined as $f(x) = x^2 + 2$ and $g(x) = 2x + 1$
Then evaluate (f o g)(x) and (g o f)(x).

c) Let $\sum = \{a, b, c\}$. Describe the languages represented by the regular expressions $r_1 = a\, b^*\, c^*$ and $r_2 = a^* + b^* + c^*$.

d) Define, and give examples of a complete graph and a bipartite graph that is not complete.

e) Write the following permutation as a product of disjoint cycle

$$\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ \\ 4 & 3 & 2 & 5 & 1 & 8 & 7 & 6 \end{pmatrix}$$

f) Draw the Hasse diagram of the poset $(S, \leq)$ where S = { 2,3,6,12,24,36} and $x \leq y$, if x divides y.

g) Write the truth table for the Boolean expression $(\sim p \rightarrow q) \vee (\sim q \rightarrow p)$.

(7x4)

2.

a) Solve the following recurrence relation:
$u_n = u_{n-1} + u_{n-2}$, $n \geq 3$, with
$u_1 = 1$, $u_2 = 3$.

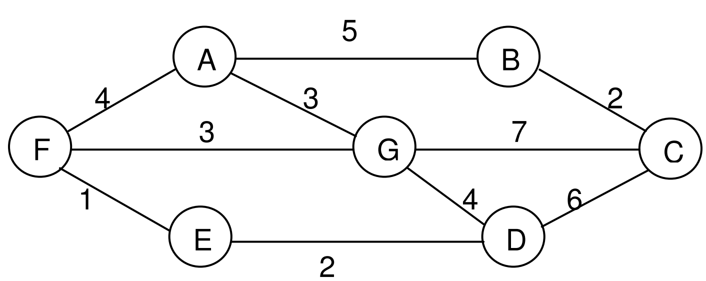
b) If $A = \begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix}$ $a, b \in R$
Then using mathematical induction prove that
$A^n = \begin{bmatrix} a^n & 0 \\ 0 & b^n \end{bmatrix}$

(10+8)

3.

a) Show that the set G = {1,2,3,4,5,6} is a finite abelian group under multiplication mod 7. Find the identify and inverse of each element.

b) Find the minimum spanning tree in the following weighted graph using Prim's algorithm. Explain the steps involved.



(8+10)

4.

a) A fair coin is tossed 10 times. What is the probability of getting at least 7 heads?

b) Write a regular expression for the language L(r) ={ w | w $\{0+1\}^*$; w has no pair of consecutive zeros}. Also construct a finite state machine for the language L(R).Write the corresponding grammar also.

c) Prove that the order of an element in a group is same as the order of its inverse.

(6+6+6)

5.

a) Using Karnaugh map, simplify the Boolean expression

$$f(x, y, z) = \sum (0,2,5,6,7)$$

b) Let A = {a, b, c} be a non empty set and R be the relation on A that has the matrix M_R

$$\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix}$$

Construct the diagram of relation R and list the in degree and out degree of all the vertices. Determine the matrices for R^2 and R^3.

c) Let I be the set of all integers and
R = {(a, b): a,b $\in$ I and a-b is divisible by 3}.
Prove that R is an equivalence relation.

(6+6+6)

6.

a) i) Draw a Hamiltonian graph with six vertices such that it is Eulerian.
ii) Draw an Eulerian graph with six vertices such it is not Hamiltonian.

b) Use pigeonhole principle to show that a set of 37 positive integers contain two numbers which have same remainder when divided by 36.

(10+8)

7.

a) Find the state diagram for the non deterministic finite state machine with state table shown below. Final state are s2 and s3 while initial state is s0, alphabet being {0,1,2}.

b) Find a deterministic finite state machine that is equivalent to the above nondeterministic finite state machine. Justify also why is it equivalent?

δ	0	1	2
S0	S0	S2	S1
S1	S3	-	S4
S2	-	-	S4
S3	S3	-	S4
S4	S3		S3

(9+9)