B4.2-R3: DISCRETE STRUCTURES

NOTE:

1.	**Answer question 1 and any FOUR from questions 2 to 7.**
2.	**Parts of the same question should be answered together and in the same sequence.**

Time: 3 Hours **Total Marks: 100**

1.

a) Show that the function f from R to R [where R is the set of real numbers] defined by

$f(x) = 3x - 1$

is invertible. Also, find the inverse of f.

b) Let A and B be two sets. Show that if A-B=A then A and B are disjoint sets.

c) Find the general expression for x_n, where $x_0 = 2$ and $x_n = 3 + x_{n-1}$, $n > 0$.

d) Let f be the permutation defined on the set {1, 2, 3, 4, 5, 6} by $f = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 2 & 1 & 5 & 6 & 3 & 4 \end{pmatrix}$.

Find f^2. Also, express f as a product of disjoint cycles.

e) Using the pigeon hole principle, find the minimum number of students in a class to be sure that atleast 4 are born in the same month.

f) In how many ways four students can be seated out of 12 if two particular students are not included at all.

g) Let G be the grammar $\{V_N, V_T, S, P\}$ where $V_N=\{S,A,B\}$; $V_T=\{a,b\}$, S is the starting symbol and

i) $G_1=\{S\rightarrow AB, A\rightarrow a, B\rightarrow bb\}$

ii) $G_2=\{S\rightarrow AB, S\rightarrow bA, A\rightarrow a, B\rightarrow ba\}$.

Find $L(G_1)$ and $L(G_2)$. (L(G) stands for the language generated by the grammar G).

(7x4)

2.

a) Among 35 students in a class, 12 study mathematics, 8 study mathematics but not english and 10 do not study any of these two subjects. Find the number of students who study english but not mathematics.

b) Let S={0, 1, 2, …, 20}. Let R be the relation defined on S by aRb if and only if a and b leave the same non-negative remainder on dividing by 3. Show that R is an equivalence relation.

(8+10)

3.

a) Let (L, ≤) be any lattice. Let a, x, y be elements of L such that x ≤ y. Show that $a \wedge x \leq a \wedge y$.

b) In S_3, find two permutations f and g that do not commute.

c) Using principle of induction, show that 133 always divides $(11)^{n+2} + (12)^{2n+1}$, where n is any positive integer.

(6+4+8)

4.

a) Let $(G_1, +)$ and $(G_2, \cdot)$ be groups where $G_1=\{0, \pm1, \pm2, \ldots\}$ and $G_2=\{2^n : n \in G_1\}$. Define a mapping f from G_1 to G_2 by $f(a) = 2^a$. Is f an isomorphism? Justify your answer.

b) Solve the congruence relation $8x \equiv 3 \pmod{27}$.

(8+10)

5.

a) Express the following Boolean function and its complement in DNF.

$$f(x, y, z) = (x \vee y) \wedge (x \vee y') \wedge (x' \vee z).$$

b) Solve the recurrence relation $a_n - 5\, a_{n-1} + 6\, a_{n-2} = 2^n + n$.

(8+10)

6.

a) Show that $((p \vee \neg q) \wedge (\neg p \vee \neg q)) \vee q$ is a tautology, where p and q are Boolean variables.

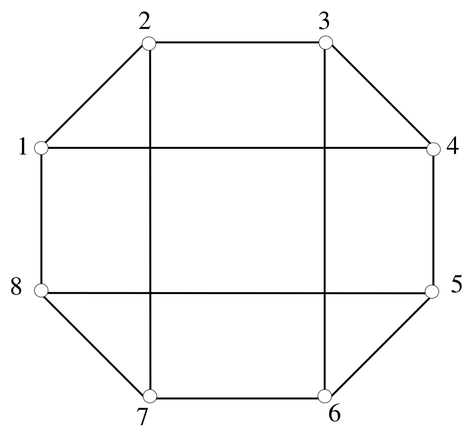
b) Let $A=\{1, 2, 3, 4\}$ and R_1 and R_2 be relations on A defined by

$R_1=\{(1, 2), (2, 3), (4, 1), (1, 4)\}$

$R_2=\{(1, 1), (2, 1),(4,1)\}$

Find $R_1 \circ R_2^{-1}$.

c) Consider the following graph



Is it a complete bipartite graph? Justify.

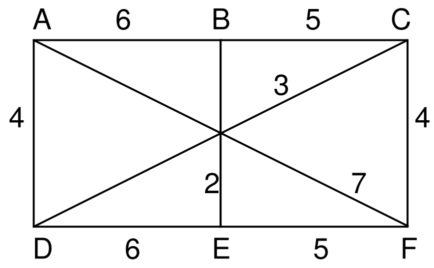
(6+6+6)

7.

a) Simplify the following Boolean function by using Karnaugh map:

$$f(x, y, z) = \sum(2, 3, 6, 7).$$

b) Determine the minimum spanning tree of the following weighted graph by using Kruskal's Algorithm.



(8+10)