C3-R4: MATHEMATICAL METHODS FOR COMPUTING

NOTE:

1.	**Answer question 1 and any FOUR from questions 2 to 7.**
2.	**Parts of the same question should be answered together and in the same sequence.**

Time: 3 Hours **Total Marks: 100**

1.

a) A fair dice is thrown 3 times. What is the probability that the number 6 will appear in at least one of the throw.

b) If events A_1 and A_2 are disjoint, then show that

$$P(A_1^c \cup A_2^c) = 1$$

c) A committee of size 8 is selected at random from 12 professional people and 28 non-professional people. What is the probability that exactly 3 members of the committee are professional people?

d) Suppose that people immigrate into a territory at a Poisson rate λ=2 per day. (i) What is the expected time until the 10^{th} immigrant arrives? (ii) What is the probability that the elapsed time between the 10^{th} and 11^{th} arrival exceeds two days?

e) Convert the following linear programming problem to the *standard form*: (do not solve it)

$$\begin{aligned} \text{maximize} \quad & 3x_1 + x_2 \\ \text{subject to} \quad & 0 \le x_1 \le 4 \\ & x_1 + x_2 \le 7 \\ & x_1 + 2x_2 \le 9 \\ & x_2 \ge 0 \end{aligned}$$

f) Find the Laplace transform of the function f(t) = Sin at.

g) Machines in a factory break down at an exponential rate of 6 per hour. A single repairman fixed the machine at an exponential rate of 8 per hour. What is the average number of broken machines in a factory?

(7x4)

2.

a) A timber merchant has three machines producing mouldings: 35% of them by Machine 1, 20% by Machine 2, 45% by Machine 3. The proportion of sub-standard moulding produced by each machine are 1%, 3% and 2% respectively.
   i) What is the overall proportion of sub-standard moulding?
   ii) What is the probability that randomly selected sub-standard moulding was produced by Machine 2?

b) At a clinic the social workers are so busy that only 60% of potential new patients that telephone are able to talk immediately with a social worker when they call. The other 40% are asked to leave their phone numbers. About 75% of the time a social worker is able to return the call on the same day and other 25% of the time the caller is contacted on the next day. Experience at the clinic indicate that the probability a caller will actually visit the clinic for consultation is 0.8 if the caller was immediately able to speak to a social worker, and it is 0.6 and 0.4, if the patient's call was returned the same day or the next day, respectively.
   i) What percentage of people that telephone visit the clinic for consultancy?
   ii) What percentage of patients that visit the telephone calls returned immediately?

(9+9)

3.

a) Show that, the Laplace Transform of

$$t^n \cosh(bt)$$

where n is a positive integer, is given by

$$\frac{n!}{2} \cdot \frac{(s+b)^{n+1} + (s-b)^{n+1}}{(s^2-b^2)^{n+1}}$$

b) Let $E_k = \frac{1}{\Pi}\int_{\Pi}^{\Pi} f_k(x)dx$ denotes the energy of the k-th harmonic mean $f_k(x) = C_k\ Coskx + b_k\ Sinkx$, Prove that $E_k = a_k^2 + b_k^2$.

(9+9)

4.

a) Solve the following linear program using Simplex method:

maximize $2x_1 + 5x_2$
subject to $x_1 \le 4$
$x_2 \le 6$
$x_1 + x_2 \le 8$
$x_1 , x_2 \ge 0$

b) Consider the linear program

minimize $15x_1 + 4x_2 + 28x_3$
subject to $5x_1 + x_2 + 7x_3 \ge 13$
$3x_1 + x_2 + 5x_3 \ge 9$
$x_1 , x_2, x_3 \ge 0$

Write the dual problem, and find the optimal solution to the dual.

(9+9)

5.

a) If 4 married couples are arranged in a row, find the probability that no husband sits next to his wife.

b) The joint density of X and Y, two continuous random variables is given by
$f(x,y) = e^{-(x+y)}$ for $0 < x < \infty$ and $0 < y < \infty$
$f(x,y) = 0$ otherwise

Find the density function of the random variable X/Y.

(9+9)

6. Let Y_1 and Y_2 have the joint probability density function given by

$$f(y_1, y_2) = \begin{cases} ky_1y_2, & 0 \le y_1 \le 1;\ 0 \le y_2 \le 1 \\ 0, & \text{elsewhere.} \end{cases}$$

a) Find the value of k that makes this a probability density function.

b) Find the joint distribution function for Y_1 and Y_2.

(10+8)

7.

a) If X and Y are independent Poisson random variables with respective parameters λ_1 and λ_2 , compute the distribution of X-Y.

b) Suppose that you have n switches labelled 1, ..., n that are all initially off. You can toggle the ith switch only if switches 1, ..., i – 2 are off and switch i – 1 is on (You can always toggle the first switch, since the requirements are vacuously satisfied). Let the action of toggling any of these switches be referred as a move. Let a_n be the number of moves you must make in order to turn on the nth switch. Find and solve a recurrence for a_n.

(9+9)