C4-R3: ALGORITHM ANALYSIS AND DESIGN

NOTE:

1. **Answer question 1 and any FOUR from questions 2 to 7.**
2. **Parts of the same question should be answered together and in the same sequence.**

Time: 3 Hours **Total Marks: 100**

1.

a) Under what circumstances the condition "P=NP" is true?

b) What do you mean by backtracking and why is it required? Why is it so called?

c) Distinguish between divide-and-conquer and dynamic programming with suitable examples.

d) When is a parallel algorithm called cost optimal?

e) Show that Knapsack problem does have a (suitably formulated) optimal structure property. Can you construct (recursively) all solutions which have the property?

f) If $f(n) = a_m n^m + a_{m-1} n^{m-1} + \ldots\ldots + a_0$ is a polynomial of degree m, then prove that $f(n) = \Theta(n^m)$.

g) What is the property that makes Kruskal's algorithm a unique greedy algorithm? Illustrate with an example.

(7x4)

2.

a) Solve the following Recurrence using recursion tree method.

$$T(n/3) + T(2n/3) + n$$

b) What is polynomial time reducibility? Illustrate with an example.

c) Differentiate among P, NP, NP-complete, and NP-hard class of problems with suitable examples.

(5+5+8)

3.

a) Write an algorithm to compute the k-th smallest element of a list of n numbers, where $k \leq n$. Determine the number of comparisons required to compute it, and deduce the time complexity of your algorithm.

b) Write the Kruskal's algorithm for computing a minimum spanning tree of a simple, connected, undirected graph G. Trace this algorithm to compute a minimum spanning tree for such a graph G that contains at least 9 vertices and 13 weighted edges.

(9+9)

4.

a) What are space complexity and time complexity of an algorithm? Differentiate between average, best and worse case time complexities.

b) Show that Euclid's algorithm for computing GCD of a pair of positive integers has all the necessary properties of an algorithm.

c) How can you count number of ones in a binary string? Show that the counting algorithms for the above problem of a binary string of length n have time complexities varied from O(n) to O(1).

(4+4+10)

5.

a) What is convex hull? Discuss Graham's algorithm for computing the convex hull for a given set of points on a plane.

b) Explain the terms `flow` and `capacity` in a network. What are meant by properly and improperly oriented edges? Discuss how flow is related to these kinds of edges.

(9+9)

6.

a) What are the differences between heuristic and approximation algorithms?

b) Design an approximation algorithm for colouring a planar graph.

c) Design a heuristic algorithm for chromatic partitioning of a simple, connected, undirected graph.

(4+7+7)

7.

a) What is the difference between the min-heap property and the binary search tree property?–Exemplify. Can the min-heap property be used to print out the keys of a binary tree of n vertices in sorted order in O(n) time?–Justify.

b) Devise a O(n+m) time algorithm for computing a component graph of a directed graph G=(V,E), where |V| = n and |E| = m. Make sure that your algorithm produces at most one edge between any pair of vertices in the component graph.

(10+8)