B0-R4: BASIC MATHEMATICS

NOTE:

1.	Answer question 1 and any FOUR from questions 2 to 7.	
2.	Parts of the same question should be answered together and in the same sequence.	

Time: 3 Hours **Total Marks: 100**

1.

a) Express the following in the form of $A + iB$

$$\frac{(a+ib)^2}{a-ib} - \frac{(a-ib)^2}{a+ib}.$$

b) Find the area enclosed by the curve $y = 4x^2$, $x = 0$, $y = 1$ and $y = 4$.

c) Find the characteristic roots of the matrix $\begin{bmatrix} 1 & 2 & 3 \\ 0 & 2 & 3 \\ 0 & 0 & 2 \end{bmatrix}$.

d) Solve the differential equation $\dfrac{dy}{dx} = \dfrac{y}{x} + ax$.

e) Evaluate $\displaystyle \lim_{x \to 0} \frac{e^x - e^{-x}}{x}$.

f) Test the convergence of the series $x + \dfrac{x^2}{2!} + \dfrac{x3}{3!} + \ldots$

g) If $x = a\cos t,\ y = a\sin t$, find $\dfrac{dy}{dx}$.

(7x4)

2.

a) Show that $x = 2$ is one of the root of $\Delta = \begin{vmatrix} x & -6 & 1 \\ 2 & -3x & x-3 \\ -3 & 2x & x+2 \end{vmatrix} = 0$.

b) Find the inverse of matrix by Gauss Elimination method $A = \begin{bmatrix} 1 & 2 & 1 \\ 3 & 2 & 3 \\ 1 & 1 & 2 \end{bmatrix}$.

(9+9)

3.

a) Find all the asymptotes of the curve $x^3 + y^3 - 3axy = 0$.

b) The function $f(x)$ is defined as

$$f(x) = 5x - 4 \quad \text{when } 0 < x \le 1$$
$$= 4x^3 - 3x \quad \text{when } 1 < x < 2$$

Discuss its continuity at $x = 1$.

c) Find the domain and range of the function $f(x) = \dfrac{1}{x^2 - 1}$.

(8+6+4)

4.

a) Evaluate $\displaystyle\int_0^{\pi} \frac{x}{a^2\cos^2 x + b^2\sin^2 x}\,dx$.

b) If $\vec{a} = 2i - j + k$ and $\vec{b} = 3i + 4j - k$, find $\vec{a}\cdot\vec{b}$, $\vec{a}\times\vec{b}$.

c) Find the lengths of major and minor axes of the ellipse $x^2 + 4y^2 = 16$.

(8+6+4)

5.

a) Find the point of local maxima and minima and corresponding maximum and minimum values of the function $(x-1)(x+2)^2$.

b) Discuss the convergence of series $\displaystyle\frac{1}{a\cdot 1^2 + b} + \frac{2}{a\cdot 2^2 + b} + \frac{3}{a\cdot 3^2 + b} + \cdots$.

c) Expand $\log(1 + e^x)$ using Maclaurin series upto four terms.

(6+6+6)

6.

a) Find the eccentricity, centre, foci, directrices and latus rectum of the hyperbola.
$$9x^2 - 16y^2 + 72x - 32y - 16 = 0$$

b) Solve the second order homogenous differential equation $x^2\dfrac{d^2 y}{dx^2} - 3x\dfrac{dy}{dx} + 4y = 2x^2$.

(9+9)

7.

a) Find the equation of the plane passing through the intersection of the planes $x + y + z = 6$ and $2x + 3y + 4z + 5 = 0$ and the points $(1,\ 1,\ 1)$.

b) Find the coordinates of the point on the curve $y = x^2 + 3x + 4$ the tangent at which passes through the origin.

c) If $a + b + c = 0$, find the characteristic roots of the matrix $A = \begin{bmatrix} a & c & b \\ c & b & a \\ b & a & c \end{bmatrix}$.

(6+6+6)