BE10-R4: APPLIED OPERATIONS RESEARCH

NOTE:

1. **Answer question 1 and any FOUR from questions 2 to 7.**
2. **Parts of the same question should be answered together and in the same sequence.**

Time: 3 Hours **Total Marks: 100**

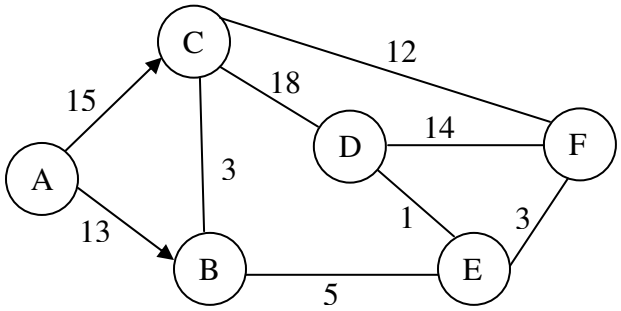
1.

a) A toy company manufactures two types of dolls. A basic version – doll ***A*** and deluxe version – doll ***B***. Each doll of type ***B*** takes twice as long to produce as one of type ***A***, and the company would have time to make a maximum of 2000 dolls per day. The supply of plastic is sufficient to produce 1500 dolls per day (both ***A*** and ***B*** combined). The deluxe version requires a fancy dress of which there are only 600 per day available. If the company makes a profit of Rs 3.00 and Rs 5.00 per doll, respectively on doll ***A*** and ***B***, then how many of each doll should be produced per day in order to maximize the total profit. Formulate the problem as L.P.P.

b) Determine the initial basic feasible solution to the following transportation problem by North-West corner rule.

	A	**B**	**C**	**Available**
I	50	30	220	1
II	90	45	170	3
III	250	200	50	4
Required	4	2	2	

c) Find the critical path for the following network:



d) Customers arrive at a sales counter manned by a single person according to a Poisson process with a mean rate of 20 per hour. The time required to serve a customer has an exponential distribution with a mean of 100 seconds per customer. Find the average waiting time of a customer in the queue and also his average waiting time in the system.

e) Formulate the following Capital Budgeting problem as a zero - one integer programming problem. There are three projects under consideration. Assume that the project run into two years. Total available funds are 80,000 (to be used at the rate of Rs 40,000/- each year). The expected profit and cost break up is as follows:

Projects	**Expected Profit**	**Cost**	
		Year 1	Year 2
1	90,000	8,000	10,000
2	60,000	2,000	5,000
3	70,000	10,000	5,000

f) Write the dual of the following linear programming problem

Maximize $Z = 2x_2 + 5x_3$

Subject to

$$x_1 + x_2 \geq 2$$
$$2x_1 + x_2 + 6x_3 \leq 6$$
$$x_1 - x_2 + 3x_3 = 4$$

and $x_1, x_2, x_3 \geq 0$

g) For the function

$$f(x, y) = \log(1 + xy), \; xy > 0,$$

find the steepest descent direction at the point $P(2,3)$.

(7x4)

2.

a) Solve the following linear programming problem by two phase method:

Minimize $Z = x_1 + x_2$

Subject to

$$2x_1 + x_2 \geq 4,$$
$$x_1 + 7x_2 \geq 7$$

and $x_1, x_2 \geq 0$

b) An automobile dealer wishes to put four repairmen to four different jobs. The repairmen have somewhat different kinds of skill and they exhibit different levels of efficiency from one job to another. The dealer has estimated the number of man hours that would be required for each job-man combination. This is given in the matrix form:

		Job			
		A	B	C	D
Man	1	5	3	2	8
	2	7	9	2	6
	3	6	4	5	7
	4	5	7	7	8

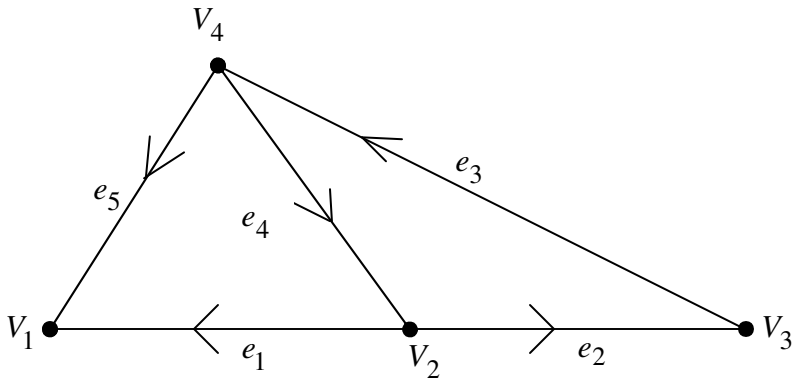
Find the optimal assignment that will result in minimum man hours needed.

(9+9)

3.

a) A company uses annually 24000 units of a raw material which costs Rs 1.25/unit. Placing each order costs Rs 22.50, and the carrying cost is 5.4 % per year of the average inventory. Find the economic lot-size, the total inventory cost, and the total annual cost including cost of material.

b) Consider the directed graph G.



Find its incidence matrix M.

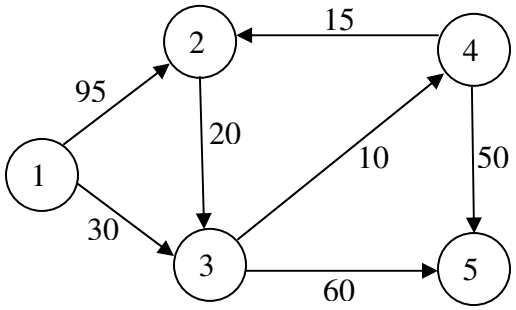
(9+9)

4.

a) Consider the following pay-off matrix. Determine the optimal strategies for player *A* and player *B* and also compute the value of the game.

$$\begin{array}{cc} & B \\ & \begin{array}{ccc} \text{I} & \text{II} & \text{III} \end{array} \\ \begin{array}{c} A \\ \\ \end{array} & \begin{array}{c} \text{I} \\ \text{II} \\ \text{III} \end{array} \begin{bmatrix} 1 & 7 & 2 \\ 6 & 2 & 7 \\ 5 & 1 & 6 \end{bmatrix} \end{array}$$

b) In the following network, five towns are connected through permissible routes. The distance (in miles) between any towns is given on the arc connecting these towns. Find the shortest distance between town 1 and any other town.



(9+9)

5.

a) A project has the following time schedule:

Activity	Time	Activity	Time
(1-2)	2	(4-8)	8
(1-4)	2	(5-6)	4
(1-7)	1	(6-9)	3
(2-3)	4	(7-8)	3
(3-6)	1	(8-9)	5
(4-5)	5		

Construct a PERT network and compute

i) Total float for each activity

ii) Critical path and time duration of the project.

b) The price breakup of a product are as follows:

Quantity	Unit Cost (in Rs.)
$0 \leq q_1 \leq 500$	10.00
$500 \leq q_2$	9.25

The monthly demand for product is 200 units, the cost of storage is 2% of the unit cost and the ordering cost is Rs. 350.00 per order. Find the optimum order quantity for a product

(10+8)

6.

a) There is congestion on the platform of a railway station. The trains arrive at the rate of 30 trains per day. The waiting time for any train to hump is exponentially distributed with an average of 36 minutes. Calculate the following:

i) Mean queue size(average no. of trains in the queue)

ii) The probability that queue size exceeds 9.

b) Solve the following game using dominance principle:

		Player B				
		I	II	III	IV	V
Player A	I	3	5	4	9	6
	II	5	6	3	7	8
	III	8	7	9	8	7
	IV	4	2	8	5	3

(9+9)

7.

a) Find all the basic solutions of the following system

$$x_1 + 2x_2 + x_3 = 4$$
$$2x_1 + x_2 + 5x_3 = 5$$

b) Five jobs go first over machine A and than over machine B in the order AB. Processing time in hours are given in the table below:

Job	Time (in hours)				
	I	II	II	IV	V
Time for A	5	1	9	3	10
Time for B	2	6	7	8	4

Determine the sequence for the six jobs that will minimize the total elapsed time.

(9+9)