C0-R4.B1: ELEMENTS OF MATHEMATICAL SCIENCES

NOTE:

1.	**Answer question 1 and any FOUR from questions 2 to 7.**
2.	**Parts of the same question should be answered together and in the same sequence.**

Time: 3 Hours **Total Marks: 100**

1.

a) Let X be a random variate with mean 8 and variance 4. Find a such that $P(X \geq a) = 0.95$.

b) Find the probability that in five tosses of a fair die the number 3 appears exactly two times.

c) Find the error in the following:

$$\lim_{x \to 1} \frac{x^2 + 3x - 4}{2x^2 - 2x} = \lim_{x \to 1} \frac{2x + 3}{4x - 2} = \lim_{x \to 1} \frac{2}{4} = \frac{1}{2}$$

What is the correct value of the first limit?

d) Evaluate $\int \frac{1}{x \ln x} dx$.

e) Find the real eigenvalue of the following:

$$A = \begin{pmatrix} 0 & 0 & 6 \\ 1/2 & 0 & 0 \\ 0 & 1/3 & 0 \end{pmatrix}$$

f) Examine the convergence of the following series

$$\frac{1}{1.2.6} + \frac{1}{2.3.7} + \frac{1}{3.4.8} + \ldots$$

g) What conic is represented by the following equation?

$$11x^2 - 4xy + 14y^2 - 58x - 44y + 71 = 0.$$

(7x4)

2.

a) A survey of 320 families with 5 children revealed the distribution shown in the following table.

Number of boys (b) and girls (g)	5b, 0g	4b, 1g	3b, 2g	2b, 3g	1b, 4g	0b, 5g	total
Number of families	18	56	110	88	40	8	320

Use chi-square test at 5% significance level to test the hypothesis that the male and the female births are equally probable ($\chi^2_{0.95} = 11.1$).

b) Without evaluating the integral and by using the mean value theorem of integral calculus, prove that

$$\frac{1}{6} < \int_0^2 \frac{1}{10 + x} dx < \frac{1}{5}.$$

(10+8)

3.

a) Solve the following system of linear equations using Cramer's rule

$$x - 3y + z = 2$$
$$3x + y + z = 6$$
$$5x + y + 3z = 3.$$

b) For any real positive x, check the convergence of the following series

$$\sum_{n=1}^{\infty} \frac{\sqrt{n}}{\sqrt{n^2+1}} \; x^n$$

(9+9)

4.

a) If $2x = y^{1/5} + y^{-1/5}$, then prove that

$$(x^2-1)\frac{d^2y}{dx^2} + x\,\frac{dy}{dx} = 25\,y.$$

b) Find the value of the determinant

$$\begin{vmatrix} 1 & a & b+c \\ 1 & b & c+a \\ 1 & c & a+b \end{vmatrix}$$

c) Prove that the following matrix is negative definite

$$A = \begin{pmatrix} -3 & 2 & 0 \\ 2 & -3 & 0 \\ 0 & 0 & -5 \end{pmatrix}$$

(10+4+4)

5.

a) For a random variable X the pdf is as follows:

$$f(x) = \begin{cases} x/2 & 0 \le x < 1 \\ 1/2 & 1 \le x < 2 \\ (3-x)/2 & 2 \le x < 3 \\ 0 & \text{Otherwise} \end{cases}$$

Find mean and variance of X.

b) Let $X_1, X_2, \ldots X_{10}$ be independent random variables, each being uniformly distributed over (0, 1). Calculate

$$P\left(\sum_{i=1}^{10} X_i > 6\right).$$

c) Let X and Y be independent Poisson variates with respective mean λ_1 and λ_2. Calculate the distribution of X+Y.

(6+6+6)

6.

a) Evaluate the integral

$$\int_0^{\pi} \frac{x \sin^3 x}{1+\cos^2 x}\, dx.$$

b) Find the equation of a circle whose diameter has (3, 4) and (1, -2) as its end points.

c) Find a vector whose length is 7 and which is perpendicular to each of the vectors $2\vec{i} - 3\vec{j} + 2\vec{k}$ and $\vec{i} + \vec{j} - \vec{k}$.

(6+6+6)

7.

a) Bag I contains 1 white, 2 black and 3 red balls; bag II contains 2 white, 1 black and 1 red balls; bag III contains 4 white, 5 black and 3 red balls. One bag is chosen randomly and two balls are drawn from it. One of the drawn balls is white and the other one is red. What is the probability that they both have been drawn from bag II?

b) It is desired to estimate the arrival rate of new calls to a cell in a mobile communication system, based on a random sample $X_1=x_1$, $X_2=x_2$, …, $X_n=x_n$ where X_i denotes the number of calls per hour in the ith observation period. Let the number of calls per hour X be Poisson distributed with parameter λ. Find maximum likelihood estimator of λ.

c) For a bivariate distribution $\{x_i, y_i\}$ i=1, …, n all the n points lie on the line y=3x+7. Find the correlation coefficient?

(6+6+6)