C4-R4: ADVANCED ALGORITHMS

NOTE:

1. **Answer question 1 and any FOUR from questions 2 to 7.**
2. **Parts of the same question should be answered together and in the same sequence.**

Time: 3 Hours **Total Marks: 100**

1.

a) What do you mean by worst case time complexity and average case time complexity? Give an example of an algorithm where they are different.

b) When do you say
   i) f(n) is O(g(n))?
   ii) f(n) is Ω(g(n))?
   Illustrate with examples.

c) Compare the efficiency of recursive and iterative algorithms.

d) Find a solution to the recurrence relation Tn = $3(T_{\frac{n}{3}})$ +c, where n is a power of 3 and c is a constant and T_1 = b.

e) Explain what you mean by saying a problem π_1 is polynomial time reducible to π_2? How is it useful in proving problem NP-complete?

f) Explain and compare the design methods 'Divide and Conquer' and 'Dynamic Programming'.

g) Give the worst case time complexity of finding if there is a cycle in directed graph of *n* nodes and *e* edges.

(7x4)

2.

a) Write a procedure for finding the maximum of n integers and then the minimum of them. How many comparisons do you need to do this? Explain how you can find maximum and minimum simultaneously with less number of comparisons?

b) A defective chessboard is a $2^k \times 2^k$ board of squares with exactly one defective square. For any k how many different defective chessboards are there? You are required to tile a defective chessboard using triominoes (L shaped polygons covering exactly 3 squares). Write an algorithm to solve this problem using divide and conquer approach.

(9+9)

3.

a) Explain the dynamic programming paradigm illustrating with the matrix-chain multiplication problem.

b) What is the smallest number of scalar multiplication required to multiply six matrices A_1, A_2, A_3, A_4, A_5, A_6 where size of each is given below.

Matrix	Dimensions
A_1	30 x 35
A_2	35 x 15
A_3	15 x 5
A_4	5 x 10
A_5	10 x 20
A_6	20 x 25

c) Explain KMP algorithm for string matching and discuss the complexity.

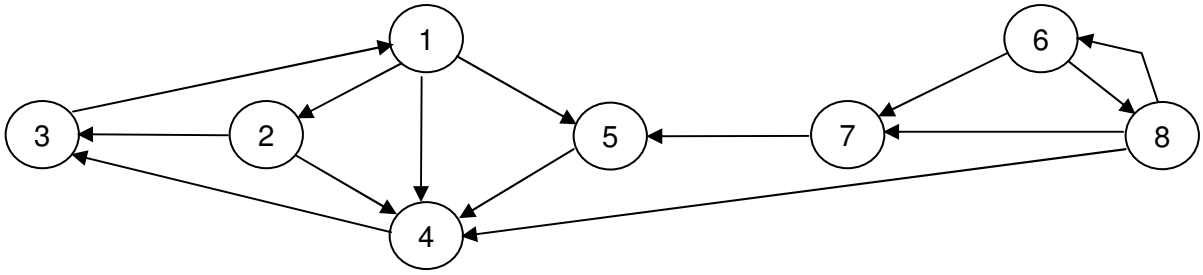
(6+4+8)

4.

a) Briefly explain DFS on a directed graph.

b) Using depth first search write an algorithm to find the strongly connected components of a graph.

c) Illustrate the working of your algorithm for the following graph.



(4+8+6)

5. Write short notes on the followings:

a) Amortized analysis.

b) RSA algorithm for public-key cryptography.

c) Approximation algorithms.

(6+6+6)

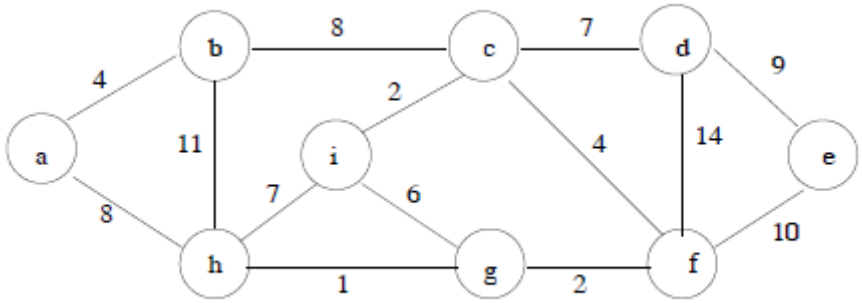
6.

a) What do you mean by a spanning tree of a graph and minimum spanning tree of a graph?

b) Describe Prim's algorithm for finding minimal spanning tree and discuss its complexity.

c) Describe Kruskal's algorithm for the same and discuss its complexity.

d) Illustrate the two algorithms step by step for the following graph.



(2+5+5+6)

7.

a) What do you mean by a NP-complete problem? Prove that Boolean satisfiability is NP-complete? Will the theorem still hold if the Boolean expression is in CNF?

b) Explain what you mean by the clique decision problem. Show that the clique problem is NP-complete.

(12+6)