

B0-R4: BASIC MATHEMATICS

NOTE:

1. Answer question 1 and any FOUR from questions 2 to 7.
2. Parts of the same question should be answered together and in the same sequence.

Time: 3 Hours

Total Marks: 100

1.

- a) Find $\lim_{x \rightarrow 0} \frac{\sin ax}{\sin bx}$.
- b) Solve the differential equation $\frac{dy}{dx} = (e^x + 1)y$.
- c) Express the matrix $\begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix}$ as the sum of a symmetric and skew symmetric matrix.
- d) Evaluate $\int_0^{\pi/2} \frac{dx}{1 + \tan^3 x}$.
- e) If $\vec{a} = 5i - 4j + k$, $\vec{b} = -4i + 3j - 2k$ and $\vec{c} = i - 2j - 7k$ then find $\vec{c} \cdot (\vec{a} \times \vec{b})$.
- f) Find the equation of the ellipse with vertices $(0, \pm 10)$ and focus $(0, \pm 8)$.
- g) Express $\frac{(1+i)(1+\sqrt{3}i)}{1-i}$ in the form of $x + iy$ where x and y are real numbers.

(7x4)

2.

- a) Use the properties of a determinant to show that $\begin{vmatrix} 0 & ab^2 & ac^2 \\ a^2b & 0 & bc^2 \\ a^2c & b^2c & 0 \end{vmatrix} = 2a^3b^3c^3$.
- b) Find the eigen values and eigen vectors of the matrix $\begin{bmatrix} a & h & g \\ 0 & b & 0 \\ 0 & 0 & c \end{bmatrix}$.
- c) Find A^{-1} if

$$A = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}$$

Prove that $A^2 - 4A - 5I = 0$.

(6+6+6)

3.

- a) A spherical soap bubble is expanding so that its radius is increasing at the rate of 0.02 centimeter per second. At what rate is the surface area increasing when its radius is 4 cm? (Take $\pi = 3.14$)
- b) Using Lagrange's mean value theorem, find a point on the curve $y = \sqrt{x-2}$ defined in the interval $[2, 3]$, where the tangent parallel to the chord joining the end points of the curve.
- c) Evaluate $\int x^2 e^{x^3} dx$.

(6+6+6)

4.

- a) Find the area of the region bounded by the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$.
- b) Find the limit when $n \rightarrow \infty$ of the series
- $$\frac{1}{n} + \frac{n^2}{(n+1)^3} + \dots + \frac{1}{8n}.$$
- c) Solve the following system of linear equations by Cramer's rule
- $$\begin{aligned} x - 4y - z &= 11 \\ 2x - 5y + 2z &= 39 \\ -3x + 2y + z &= 1 \end{aligned}$$

(6+6+6)

5.

- a) Discuss the continuity of the function at $x = 0$, where
- $$f(x) = \begin{cases} 3x - 2, & \text{when } x \leq 0 \\ x + 1, & \text{when } x > 0 \end{cases}.$$
- b) Test the convergence of the series, whose general term is $\frac{n+1}{n^3} x^n$.
- c) Solve the differential equation $\frac{dy}{dx} = \frac{x^2 - y^2}{xy}$.

(6+6+6)

6.

- a) Find the rank of the matrix $\begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ 3 & 5 & 7 \end{bmatrix}$.
- b) If $f'(x) = 3x^2 - \frac{2}{x^3}$ and $f(1) = 0$, find $f(x)$.
- c) Find the eccentricity, coordinates of foci and length of the latus rectum of the ellipse $4x^2 + 3y^2 = 36$.

(6+6+6)

7.

a) Find the locus of Z , given that $\left| \frac{Z-3}{Z+3} \right| = 2$.

b) Find the local maxima and minima of the function

$$f(x) = \sin x - \cos x, \quad \text{where } 0 < x < \frac{\pi}{2}.$$

c) Find the equation of the line having slope 5 and which divides the join of (5, 10) and (13, 6) in the ratio 2:3 internally.

(6+6+6)