C0-R4.B1: ELEMENTS OF MATHEMATICAL SCIENCES

NOTE:

1.	**Answer question 1 and any FOUR from questions 2 to 7.**
2.	**Parts of the same question should be answered together and in the same sequence.**

Time: 3 Hours **Total Marks: 100**

1.

a) Obtain the matrix multiplication *AB, B'A'* of matrices A=$\begin{bmatrix} 4 & 3 \\ 7 & 2 \\ 9 & 0 \end{bmatrix}$ and B=$\begin{bmatrix} 2 & 5 \\ 1 & 6 \end{bmatrix}$, where A' denotes transpose of a matrix A.

b) Evaluate: $\lim_{x \to 0} \frac{x - \sin x}{x^3}$.

c) Find the equation of the circle which passes through the points (1, 0), (0, –1) and (1, 1).

d) Using integral test, determine the convergence of the series: $\sum_{n=2}^{\infty} \frac{1}{n \log n}$.

e) Solve the following system of linear equations by the Cramer's rule:

-2x + 3y – 8 = 0

3x – y + 5 = 0.

Also show the detailed working of the rule.

f) State if the following statement is True or False with proper reasoning

"Mean of a binomial distribution is 3 and variance is 4."

g) Let the roots of the quadratic equation $x^2 - 7x + 15 = 0$ be α and β. Find the quadratic equation where roots are α^2 and β^2.

(7x4)

2.

a) If the coefficient of correlation between two variates X and Y is 0.3, the covariance is 9, and the variance of X is 16, then find the standard deviation of Y.

b) Find the equation of a parabola having origin as its vertex, the y-axis as the axis of parabola and it passes through the point (-10, -5).

c) Find semi major axis, semi minor axis and eccentricity of the ellipse given by $\frac{x^2}{4} + \frac{y^2}{9} = 1.$

(6+6+6)

3.

a) Find the sum of the series: $\sum_{n-1}^{\infty} \frac{4}{2^n}$.

b) Examine whether the following system of linear equations is consistent or not.

$x + 2y = 3$
$y - z = 2$
$x + y + z = 1$.

c) Find the eigen values of the matrix $\begin{bmatrix} 1 & -2 \\ -5 & 4 \end{bmatrix}$.

(6+6+6)

4.

a) Find two positive numbers such that their arithmetic mean is 12.5 and geometric mean is 10.

b) Evaluate the following integrals:

i) $\int \frac{x^4 - 1}{x - 1} dx$

ii) $\int \frac{2x + 5}{x^2 + 5x + 5} dx$

(6+12)

5.

a) Test the convergence of the following infinite series:

i) $\sum_{n=1}^{\infty} \frac{n!}{n^n}$

ii) $\sum_{n=1}^{\infty} \frac{1}{(n+1)^2}$

b) Using Taylor series, expand $f(x) = 11x^3 + 43x^2 - 60x + 14$ in powers of $(x - 3)$.

(10+8)

6.

a) From the following data compute the Arithmetic Mean of marks of the class:

Marks	**0-10**	**10-20**	**20-30**	**30-40**	**40-50**	**50-60**
No. of Students	5	10	25	30	20	10

b) Verify the Lagrange's Mean Value Theorem for the function $f(x) = 2x^2 + 3x - 5$ in the interval [-1, 1].

c) Find the probability of getting exactly 4 heads in 6 tosses of a fair coin.

(6+6+6)

7.

a) Find the regression line of y on x for the following data:

x	1	4	2	3	5
y	3	1	2	5	4

b) Assume that the probability of an individual coal miner being killed in a mine accident during an year is $\frac{1}{2400}$. Use Poission distribution to calculate the probability that in a mine employing 200 minors, there will be at least one fatal accident in a year.

(10+8)