C0-R4.B1: ELEMENTS OF MATHEMATICAL SCIENCES

NOTE:

> 1. Answer question 1 and any FOUR from questions 2 to 7.
> 2. Parts of the same question should be answered together and in the same sequence.

Time: 3 Hours | **Total Marks: 100**

1.

a) If α, β are the roots of the equation $4x^2 + 3x + 7 = 0$, then find the value of $\dfrac{1}{\beta} + \dfrac{1}{\alpha}$.

b) Evaluate $\displaystyle \lim_{x \to 0} \frac{\log \sin 2x}{\log \sin x}$.

c) Test the convergence of the series $\displaystyle \sum_{n=1}^{\infty} \frac{(n+1)!}{3^n}$.

d) Evaluate $\displaystyle \int \frac{1}{(x-1)(x^2+1)} dx$.

e) If $\vec{a} = i - 2j + k, \vec{b} = 2i + j + k$ and $\vec{c} = i + 2j - k$ then determine the vector $\vec{a} \times (\vec{b} \times \vec{c})$.

f) Find the equation of the circle whose centre is $(1, -3)$ and which touches the line $2x - y - 4 = 0$.

g) If $\begin{bmatrix} 4 & 1 & 2 \\ 0 & 5 & 3 \end{bmatrix} \begin{bmatrix} 3 & 4 \\ -1 & 0 \\ 3 & 4 \end{bmatrix} = \begin{bmatrix} 8x + y & 6z \\ 2x & 12 \end{bmatrix}$, then find the values of x, y and z.

(7x4)

2.

a) Investigate the values of λ and μ so that the system of linear equations

$$2x + 3y + 5z = 9$$
$$7x + 3y - 2z = 8$$
$$2x + 3y + \lambda z = \mu$$

may have (i) no solution (ii) unique solution (iii) an infinite number of solutions.

b) Find the area of the region bounded by the curves $y^2 = x$ and $x^2 + y^2 = 4x$.

c) Find the maximum value of f(x) = sin x(1+cos x) in [0, π]. Show your steps clearly.

(8+6+4)

3.

a) Using Poisson distribution, find the probability that the ace of spade will be drawn from a pack of well-shuffled cards at least once in 104 consecutive trials [Given $e^{-2} = 0.135$].

b) Evaluate $\displaystyle \int_0^{\pi/2} \frac{\sin x}{\sin x + \cos x} dx$.

c) Classify if the following curve is a parabola, ellipse or hyperbola? Give reasons.
$$17x^2 - 12xy + 8y^2 + 46x + 28y + 17 = 0.$$

(6+6+6)

4.

a) Evaluate $\int \dfrac{(x+1)}{x\left(1+xe^x\right)^2}\,dx$.

b) Find all asymptotes of $x^3 + 3x^2y - 4y^3 - x + y + 3 = 0$.

(9+9)

5.

a) Test the convergence of an infinite series

$$\frac{x}{1\cdot 2} + \frac{x^2}{2\cdot 3} + \frac{x^3}{3\cdot 4} + \cdots\cdots\cdots\cdots\cdots$$

b) Find the expansion of $\log\left(1+e^x\right)$ in powers of x by Maclaurin's theorem.

c) An urn contains nine balls, two of which are red, three blue and four black. Three balls are drawn from the urn at random. What is the probability that all three balls are of same colour?

(6+6+6)

6.

a) Find the eigen values and eigen vectors of the matrix $\begin{bmatrix} 1 & 0 & -1 \\ 1 & 2 & 1 \\ 2 & 2 & 3 \end{bmatrix}$. .

b) Using Least Square Method, obtain the regression of Y on X from the following data:

X	:	1	2	3	4	5
Y	:	2	4	5	3	6

(9+9)

7.

a) Find the focus, vertices and eccentricity of ellipse $16x^2 + 25y^2 = 400$.

b) Find the coefficients of x^{10} in the expansion of $\left(2x^2 - \dfrac{1}{x}\right)^{20}$.

c) The amount in the pockets of 5 students are Rs. 30, 80, 110, 120 and 200 respectively. Find second moment about origin.

(6+6+6)