B0-R4: BASIC MATHEMATICS

NOTE:

> 1. **Answer question 1 and any FOUR from questions 2 to 7.**
> 2. **Parts of the same question should be answered together and in the same sequence.**

Time: 3 Hours **Total Marks: 100**

1.

a) Find the smallest integer *n* for which $\left(\dfrac{1+i}{1-i}\right)^n = 1$.

b) Evaluate $\lim\limits_{x \to 0} (\cos x)^{\cot^2 x}$.

c) Find the inverse of a matrix $A = \begin{bmatrix} 3 & -3 & 4 \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{bmatrix}$.

d) Test for convergence the series $\sum\limits_{n=1}^{\infty} \dfrac{\left(n + \sqrt{n}\right)}{2^n \, n^{n+1}}$.

e) Solve the differential equation $x + y\left(\dfrac{dy}{dx}\right) = 2y$.

f) Find the value of **a** such that the vectors $2i - j + k$, $i + 2j - 3k$ and $3i + aj + 5k$ are coplanar.

g) Evaluate $\displaystyle\int_0^{\pi/2} \dfrac{\sin x}{\sin x + \cos x}\, dx$.

(7x4)

2.

a) Using Matrix method, show that the equations
$3x + 3y + 2z = 1,\ x + 2y = 4,\ 10y + 3z = -2$ and $2x - 3y - z = 5$ are consistent and hence obtain the solution for *x, y* and *z*.

b) Find the equation of straight line passing through (-2, 1) and perpendicular to the line 3x + 2y + 4 = 0.

c) Find the cosine and sine of the angle between the two vectors $a = 3i + j + 2k$ and $b = 2i - 2j + 4k$. Also, find a unit vector perpendicular to both **a** and **b**.

(6+6+6)

3.

a) Evaluate $\displaystyle\int \dfrac{(x+1)}{x\left(1 + xe^x\right)^2}\, dx$.

b) Find the minimum value of $x^2 + y^2 + z^2$ when x + y + z = 2.

c) Find the asymptotes of the curve $x^3 + y^3 = 3axy$.

(6+6+6)

4.
a) Find the area bounded by the parabola $x^2 = 16y$ and its latus rectum.
b) Using Maclaurin's Theorem, approximate the function $f(x) = x \sin x$ about $x = 0$ by a third degree polynomial in x.

c) If $p^2 = a^2 \cos^2 \theta + b^2 \sin^2 \theta$, prove that $p + \dfrac{d^2 p}{d\theta^2} = \dfrac{a^2 b^2}{p^3}$.

(6+6+6)

5.

a) Verify Rolle's Theorem in the interval $[-3, \ 0]$ for the function $f(x) = x(x+3)e^{-x/2}$.

b) Test for convergence the series $\sum \dfrac{x^n}{1 + \sqrt{n}}$.

c) Solve the differential equation $\dfrac{d^2 y}{dx^2} - 4\dfrac{dy}{dx} + 3y = e^{2x} \sin 3x$.

(6+6+6)

6.

a) Let $\dfrac{d}{dx} F(x) = \dfrac{e^{\sin x}}{x}, x > 0$, and if $\displaystyle\int_1^4 \dfrac{2e^{\sin x^2}}{x} dx = F(k) - F(1)$ then find the possible value of k.

b) Find the equation of the parabola whose focus is the point $(0,0)$ and whose directrix is the straight line $3x - 4y + 2 = 0$.

c) Find the centre, the length of the axes and the eccentricity of the ellipse
$$2x^2 + 3y^2 - 4x - 12y + 13 = 0$$

(6+6+6)

7.

a) Find the Eigen values and Eigen vectors of the matrix $\begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix}$.

b) Find the values of constants a and b such that
$$\lim_{x \to 0} \dfrac{axe^x - b\log(1+x) + x}{x^3} = \dfrac{-1}{18}$$

c) Show that the curves $x^3 - 3xy^2 = -2$ and $3x^2 y - y^3 = 2$ cut orthogonally.

(6+6+6)