

## BE10-R4: APPLIED OPERATIONS RESEARCH

**NOTE:**

1. Answer question 1 and any FOUR from questions 2 to 7.
2. Parts of the same question should be answered together and in the same sequence.

**Time: 3 Hours**

**Total Marks: 100**

**1.**

- a) Determine the dual of the following program with all dual variables non-negative only.

$$\begin{aligned} \text{Maximize: } z &= x_1 + 3x_2 - 2x_3 \\ \text{subject to: } 4x_1 + 8x_2 + 6x_3 &= 25 \end{aligned}$$

$$7x_1 + 5x_2 + 9x_3 \leq 30$$

- b) Is the set of vectors  $\{ [1, 1, 3, 1]^T, [1, 2, 1, 1]^T, [1, 0, 0, 1]^T \}$  linearly independent? Justify.

- c) Determine whether the vector  $[1, 2, 3]^T$  is a linear combination of the vectors

$$[1, 2, 1]^T, [1, 1, 1]^T, [2, 3, 2]^T.$$

- d) Suppose the passengers arrive at a railway reservation office at the rate of 25 per hours. If the approximate averaging time of issuing ticket is 4 min, how many counters shall be opened in the least so that the queue does not go on increasing indefinitely?

- e) A wholesaler supplies 20 tins of special food each weekday to various customers. Tins are purchased from a manufacturer in lots of 100 each for Rs. 1000 per lot. Each order placed incurred a handling charge of Rs. 50 and freight charge of Rs. 200 per lot. The incremental cost is Rs. 0.50 per year per tin for inventory. Assuming 250 workdays in an year, find economic order to minimize total inventory cost.

- f) Six jobs are to be processed on two machines A and B in order. The time required to process these jobs is described below:

Job	1	2	3	4	5	6
<b>Machine A (hours)</b>	30	120	50	20	90	110
<b>Machine B (Hours)</b>	80	100	90	60	30	10

Find the job sequence to minimize the overall process time of all six jobs.

- g) Perform one complete step of the steepest descent method on the problem

$$\text{Min } f(x, y) = x^2 + 3y^2 - 4x - 6y + 7$$

Starting from the point (1, 1).

**(7x4)**

**2.**

- a) The men's department of large store employs one tailor for customer fittings. The number of customers requiring fitting appears to follow a Poisson distribution with mean arrival rate 24 per hour. Customers are serviced on first come first served basis and they are always willing to wait for the tailor's service, because alternations are free. The time it takes to fit a customer appears to be exponentially distributed with a mean of 2 min. (i) what is the average number of customers in the fitting room? (ii) How much time should a customer expect to spend in the fitting room? (iii) What percentage of the time is the tailor idle?

- b) Consider the following LP problem:

$$\text{Maximize } z = 3x_1 + 2x_2 + 4x_3$$

$$\text{subject to } 2x_1 + 3x_2 - x_3 \leq 12$$

$$x_1 + x_2 + 2x_3 \leq 10$$

$$x_1, x_2, x_3 \geq 0.$$

The optimal simplex table for the standard form of the above problem with  $x_4$  and  $x_5$  as slack variables

		$x_1$	$x_2$	$x_3$	$x_4$	$x_5$	
		3	2	4	0	0	
$x_1$	3	1	1.4	0	0.4	0.2	6.8
$x_3$	4	0	-0.2	1	-0.2	0.4	1.6
$(z_j - c_j)$ :		0	-1.4	0	0.4	2.2	26.8

If the objective function is changed to maximize  $z = x_1 + 5x_2 + 8x_3$ , then find the new optimal solution through sensitivity analysis.

(9+9)

3.

a)

Construct a network diagram for a project consisting of the following activities:

Activity	Immediate Predecessor(s)	Optimistic(a)	Most Likely(m)	Pessimistic(b)
A	-	1	3	5
B	-	3	4	5
C	A,B	4	5	6
D	B	3	5	7
E	D	5	6	13

Determine the expected completion time and variance for the project.

b)

A company is faced with the problem of assigning six different machines to five different jobs. The costs are estimated as follows (in thousands):

2.5	5	1	6	1
2	5	1.5	7	3
3	6.5	2	8	3
3.5	7	2	9	4.5
4	7	3	9	6
6	9	5	10	6

Solve the problem assuming that the objective is to minimize total cost.

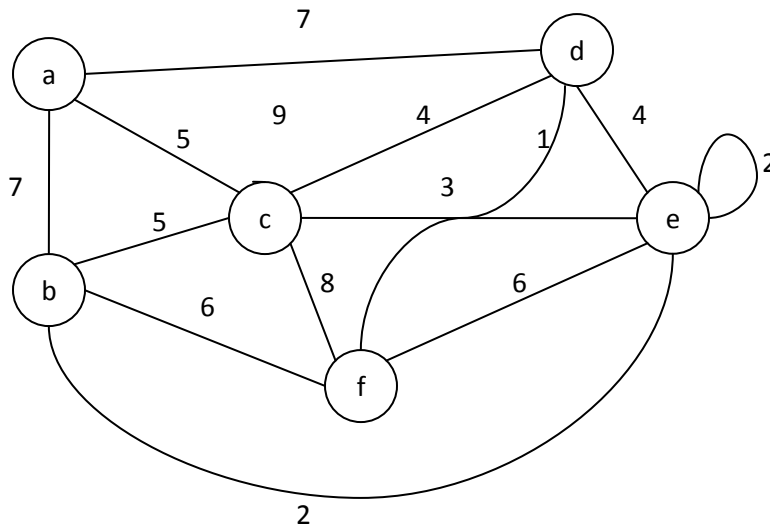
(10+8)

4.

a) Determine the optimal transportation plan in the following table giving plants to markets shipping costs, with three plants and four markets.

	M1	M2	M3	M4	availability
P1	11	20	7	8	50
P2	12	9	4	7	40
P3	8	12	18	9	70
Requirement	30	25	35	40	

b) Find the minimum spanning tree of the following graph using Kruskal's method. Explain the steps involved in it clearly.



(9+9)

5.

a) Consider an inventory system with the following data: annual demand for a particular item is 1500 units; carrying cost of one unit is Rs.15.00; ordering cost is Rs. 15.00. Determine the following:

- i) Economic order quantity
- ii) number of orders per year
- iii) total inventory cost per year

- b) Use Dijkstra's algorithm to find the length of a shortest path between vertices no 1 and no 6 in the weighted graph represented in the following matrix, where  $\infty$  denote that the edge in graph is not present.

	1	2	3	4	5	6
1	0	4	2	$\infty$	$\infty$	$\infty$
2	4	0	1	5	$\infty$	$\infty$
3	2	1	0	8	10	$\infty$
4	$\infty$	5	8	$\infty$	2	6
5	$\infty$	$\infty$	10	2	0	
6	$\infty$	$\infty$	$\infty$	6	3	0

(9+9)

6.

- a) We have five jobs, each of which must go through the machines A, B, C in the order ABC. Processing times are given in the following table:

Job.	A	B	C
1	4	5	8
2	9	6	10
3	8	2	6
4	6	3	7
5	5	4	11

Determine a sequence for the five jobs that will minimize the elapsed time T. Also find out the minimum elapsed time.

- b) The pay off matrix of the zero sum game between two players A and B is given below. Find the solution of the game.

		B				
		I	I	III	IV	V
A	I	-2	0	0	5	3
	II	3	2	1	2	2
	III	-4	-3	0	-2	6
	IV	5	3	-4	2	-6

(10+8)

7.

a) Use Branch and Bound technique to solve the following Integer Linear programming problem:

$$\text{Max. } Z = 7x_1 + 9x_2$$

subject to

$$-x_1 + 3x_2 \leq 6$$

$$7x_1 + x_2 \leq 35$$

$0 \leq x_1, x_2 \leq 7$ ;  $x_1$  and  $x_2$  are integers.

b) Draw a network diagram from the following activity and find critical path and total duration of the project:

Activity	Duration (days)
1-2	2
1-3	2
1-4	1
2-5	4
3-6	5
3-7	8
4-7	3
5-8	1
6-8	4
7-9	5
8-9	3

(9+9)