C3-R4: MATHEMATICAL METHODS FOR COMPUTING

NOTE:

1.	**Answer question 1 and any FOUR from questions 2 to 7.**
2.	**Parts of the same question should be answered together and in the same sequence.**

Time: 3 Hours **Total Marks: 100**

1.

a) Find Laplace of $\mathcal{L}\left(e^{2t}+4t^3-2\sin 3t\right)$.

b) Find Fourier coefficient a_n and b_n of the following function:

$$f(x)\begin{cases}0,-\pi\leq x<0\\ \pi,0\leq x\leq\pi\end{cases}.$$

c) A biased coin with probability p(0<p<1) of success (head) is thrown until for the first time same results occur three times in succession (either 3 heads or 3 tails). Find the probability that the game ends in seventh throw.

d) A Manufacturer produces two different products, X_1 and X_2 using three machines: M_1, M_2, M_3. Each machine can be used only for a limited time. Production times of each product on each machine are given in the following Table.

	Production time (in hours/unit)		Available time
Machine	X_1	X_2	(hours)
M_1	1	1	8
M_2	1	3	18
M_3	2	1	14
Total	4	5	

The objective is to maximize the combined production time for utilizing the three machines. Formulate this problem as a linear programming problem.

e) Write the dual program of the following linear programming problem:

Min $z=5x_1-3x_2$

subject to constraints

$x_1-x_2\leq 9$

$3x_1+5x_2\geq 11$

$x_1\geq 0, x_2$ *is unrestricted*

f) Solve the following matrix game:

		B			
		1	2	3	4
A	1	4	6	-2	1
	2	3	3	4	2
	3	4	5	5	1

and find optiomal strategies for both player A and player B.

g) Six fair coins are tossed 6400 times. Using the binomial distribution, find the mean of getting six heads.

(7x4)

2.

a) Measurements at the Universities Computation Centre (UCC) on a certain day indicated that the source of incoming jobs is 15 percent from Delhi, 35 percent from Mumbai and 50 percent from Bangalore. Suppose that the probabilities that a job initiated from these cities require set-up are 0.01, 0.05 and 0.02 respectively. Find the probability that a job chosen at random at UCC is a set-up job. Also find the probability that a randomly chosen jobs comes from Bangalore that it is a set-up job.

b) Assume that the life time X and the brightness Y of an LCD bulb are being modeled as continuous random variables. Let the joint probability density function be given by

$$f(x, y) = \lambda_1 \lambda_2 e^{-(\lambda_1 x + \lambda_2 y)},\ 0 < x < \infty,\ 0 < y < \infty .$$

Find the marginal density of X, marginal density of Y and joint distribution function of (x, y).

(8+10)

3.

a) Use the Simplex method to solve the problem

maximize $Z = 5x_1 + 4x_2$

subject to: $4x_1 + 5x_2 \le 10,\ 3x_1 + 2x_2 \le 9,\ 8x_1 + 3x_2 \le 12,\ x_1, x_2 \ge 0$

b) An arrival rate at a telephone booth is considered to be Poisson with an average time of 10 minutes and exponential call length averaging 3 minutes.

i) Find the fraction of a day that the telephone will be busy.

ii) What is the probability that an arrival at the booth will have to wait?

iii) What is the probability that an arrival will have to wait more than 10 minutes before the phone is free?

iv) What is the probability that it will take him more than 10 minute altogether to wait for phone and to complete his call?

(10+8)

4.

a) Using the KKT conditions for the following non linear programming problem verify that $x^k = \left(\frac{1}{2}, \frac{3}{2}\right)^T$ is its optimal solution

minimize $f(x) = (x_1 - 1)^2 + x_2 - 2$

subject to $h(x) = x_2 - x_1 - 1 = 0,$

$g(x) = x_1 + x_2 - 2 \le 0 .$

b) A market survey is made on two brands of breakfast foods A and B. Every time a customer purchases, he/she may buy the same brand or switch to another brand. The transition probabilities are given in table.

To From	Brand A	Brand B
Brand A	0.8	0.2
Brand B	0.6	0.4

At present, it is estimated that 60% of the people buy brand A and 40% buy brand B. determine the market share of the brands after one time period, after two time period and in the steady state.

(8+10)

5.

a) Calculate the steady state probabilities form the transition probability matrix given in table when the system can either start in state 1 or state 2

To From	State 1	State 2
State 1	0.8	0.2
State 2	0.4	0.6

b) A petrol station has a single pump and space for not more than three cars (two waiting, one being served). A car arriving when the space is filled to its capacity goes elsewhere for petrol. Cars arrive according to Poisson distribution at a mean rate of every 8 minutes. Their service time has an exponential distribution with a mean of 4 minutes. The owner has the opportunity of renting an adjacent piece of land which would provide space for an individual car to wait. The rent would be Rs. 10 per week. The expected net profit from each customer is Rs. 0.50 and the station is open for 10 hours every day. Would it be profitable to rent the additional space? Give appropriate reasons.

(10+8)

6.

a) Find an optimal solution of the following linear integer programming problem using Branch and Bound method.

Maximize $z = 6x_1 + 8x_2$

subject to the constraints:

$4x_1 + 16x_2 \le 32$

$14x_1 + 4x_2 \le 6$

$x_1, x_2 \ge 0$ are integers.

b) Use Dynamic programming to Maximize $Z = y_1 y_2$, subject to the constraints $y_1 + y_2 = 5$ and $y_j \ge 0; j = 1,2$.

(12+6)

7.

a) Evaluate Laplace inverse, $L^{-1}\left(\frac{1}{\left(s+\sqrt{2}\right)\left(s-\sqrt{3}\right)}\right)$.

b) Find the Fourier transform of the function $f(t) = \begin{cases} k, & 0 < t < a \\ 0, & otherwise \end{cases}$.

c) An urn contains 5 white and 4 red balls. Four balls are transferred to second urn (which earlier is empty). A ball is drawn from this urn and it happens to be black. Find the probability of drawing a white ball among the remaining three from the second urn?

(6+6+6)