C4-R4: ADVANCED ALGORITHMS

NOTE:

1. **Answer question 1 and any FOUR from questions 2 to 7.**
2. **Parts of the same question should be answered together and in the same sequence.**

Time: 3 Hours **Total Marks: 100**

1.

a) Define the main characteristics of an algorithm. Determine the total computation time for the following:

```
For i = 2 to m-1
{
      For j = 3 to i
      {
      Sum = sum +A[i][j]
      }
}
```

b) State Master's theorem for solving recurrence. Consider the following recurrence:

$T(n) = 2\,T(n/4) + \sqrt{n}$

Find its asymptotic bound.

c) Sort the following list in increasing order using quick sort technique, showing all the iterations and determine the worst case running time of this method:

38,57,63,3,31,65,72,46,16,92

d) i) Arrange the functions below from lowest asymptotic order to highest asymptotic order:

2^n, n^2, n^3 ,nlog n, $\log^5 n$, n^2+ log n.

ii) Is $2^{n+1} = O(2^n)$? Is $2^{2n} = O(2^n)$? Justify.

e) Define Fibonacci heap along with its applications. Differentiate between Binomial heap and Fibonacci heap.

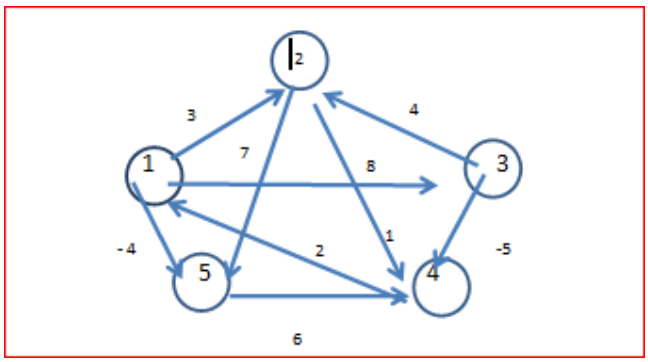
f) Prove that the height of a heap with n nodes is equal to [$\log_2 n$].

g) Describe any one linear sorting algorithm and analyse its time complexity in best and worst cases. Is it a stable sort? Justify your answer.

(7x4)

2.

a) Write the assumptions made in Floyd Warshall algorithm. Apply Floyd Warshall algorithm for constructing shortest paths. Also, show the matrices $D^{(5)}$ and $\pi^{(5)}$ computed by the Floyd Warshall algorithm for the following graph given below:



b) What is amortized analysis? A sequence of n operations is performed on a data structure. The I th operation costs I rupees if I is an exact power of 2 and 1 rupee otherwise.
Use aggregate analysis or any other to determine the amortized cost per operation. How is it different than worst case analysis?

c) Let g(n) and g(n) be asymptotically positive functions.Prove or disprove each of the following:

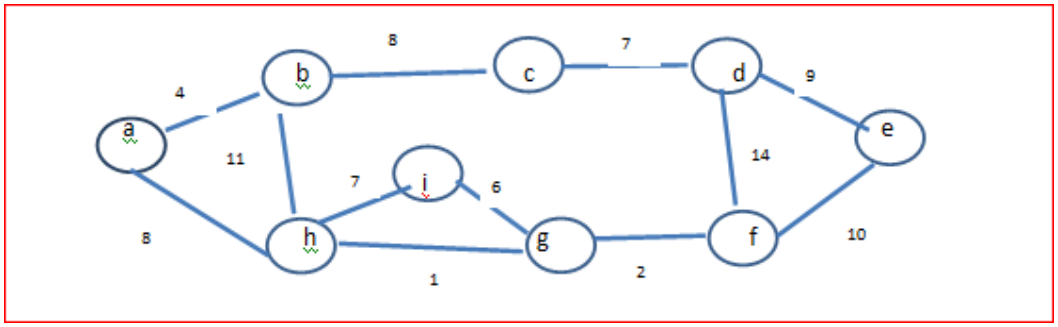
i) f(n) = O(g(n)) implies g(n) = O(f(n))

ii) f (n) =O(g(n)) implies g(n) = Ω(f(n))

(8+4+6)

3.

a) Define a spanning tree of a graph. Does every graph have a spanning tree? Let (u,v) be a minimum weight edge in a graph G, prove that (u,v) belongs to some minimum spanning tree of G.Find the minimum spanning tree of the following graph using Kruskal's algorithm. Also determine the time complexity of this algorithm.

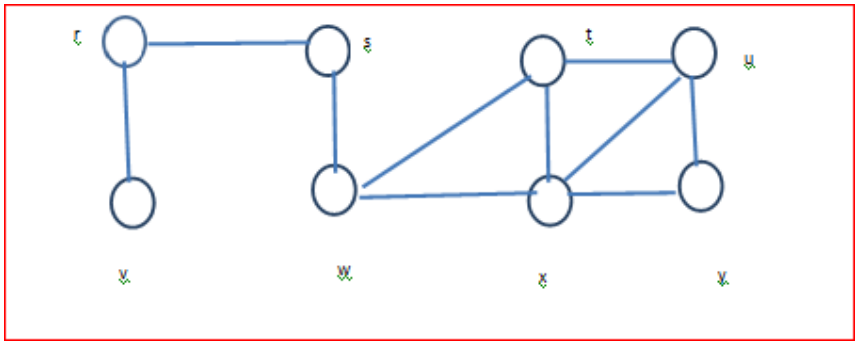


b) Upon which algorithmic approach determination of Huffman code is based? What do you understand by prefix code? Is Huffman code a prefix code? Justify your answer. Find an optimal Huffman code for the following set of frequencies:

a:50 b: 25 c:15 d:40 e: 75

Determine the code for 'abbabbced'.

c) Consider the graph G shown below. Describe the whole process of breadth first search starting with node u .Also write applications of Breadth first search.



(8+5+5)

4.

a) Explain string matching problem. Write the Knuth-Morris (KMP) algorithm. Determine its time complexity. Compute the prefix function π for the pattern

"ababbabbabbababbabb" when alphabet is Σ={a,b}.

b) Explain the difference between **any two**:

i) merge sort and quick sort

ii) prim and kruskal algorithms

iii) accounting and potential methods

c) Explain matrix chain multiplication problem. Parenthesise the following chain of matrices for optimal multiplication

A B C D E where size of these matrices are

4x10, 10x3, 3x12, 12x20, 20x7

(8+[2.5x2]+5)

5.

a) State following problems along with name one algorithm which can be used for :
   i) Set cover problem
   ii) Binpacking problem
   iii) closest pair problem
   iv) Activity selection problem

b) Write mathematical formulation of 0-1 knapsack problem. Does greedy approach work to solve it? Justify your answer.
   Can this problem be solved using Dynamic programming approach?

c) What is reducibility of one problem to another? What do you mean by NP completeness? Give examples of at least one problem of each of the classes: class P, class NP, class NP hard and class NP complete.

(8+5+5)

6.

a) Discuss Boyer-Moore algorithm for string matching.

b) Suppose that a counter begins at a number with b 1's in its binary representation, rather than at 0. Show that the cost of performing n INCREMENT operations is $O(n)$ *if* $n = \Omega(b)$. (Do not assume that b is constant).

c) Write short note on Approximation Algorithm.

(6+6+6)

7.

a) If $P \neq NP$ then prove that for any constant $\rho \geq 1$, there is no polynomial-time approximation algorithm with approximation ratio ρ for the general traveling-salesman problem.

b) State and prove the Euler theorem.

c) What is dynamic programming? Why it is called so? Solve the knapsack problem of five objects, whose weights are respectively 1, 2, 4, 5 and 7 units, and whose values are 1, 4, 6, 12 and 16. The maximum of 8 units of weight can be carried.

(6+6+6)