# B0-R4: BASIC MATHEMATICS

**NOTE:**

> 1. Answer question 1 and any FOUR from questions 2 to 7.
> 2. Parts of the same question should be answered together and in the same sequence.

**Time: 3 Hours**                                                     **Total Marks: 100**

**1.**

a)     Find Re(z) and Im(z) where $z = \dfrac{1+7i}{(2-i)^2}$ .

b)     If $y = a\sin x + b\cos x$, prove that $y^2 + \left(\dfrac{dy}{dx}\right)^2 = a^2 + b^2$

c)     If $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ and $E = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$ find the value of $(2I + 3E)^3$ .

d)     Evaluate the integral $\displaystyle\int \dfrac{x + \sin x}{1 + \cos x} dx$ .

e)     Find the first three non-zero terms of the Maclaurin's series for $f(x) = \log(1+x)$ at $x=0$.

f)     Find the equation of the parabola whose focus is $(1,-1)$ and whose vertex is $(2,1)$.

g)     Test the convergence of the series $\displaystyle\sum_{n=1}^{\infty} \dfrac{(n+1)!}{2^n}$ .

                                                                            **(7x4)**

**2.**

a)     Reduce the matrix $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ 3 & 5 & 7 \end{bmatrix}$ to its normal form and find its rank.

b)     Let the matrix $A$ be defined as $A = \begin{bmatrix} 1 & 2 & -3 \\ 0 & 3 & 2 \\ 0 & 0 & -2 \end{bmatrix}$. Find the eigen values of

        $3A^3 + 5A^2 - 6A + 2I$ .

                                                                            **(8+10)**

**3.**

a)     If $\sin y = x\sin(a + y)$, then $\dfrac{dy}{dx} = A\sin^2(a + y)$. Find $A$ and then find the value of $\dfrac{dy}{dx}$ at $x = 0$.

b)     Show that $\sin x(1 + \cos x)$ has a maximum value when $x = \dfrac{\pi}{3}$.

c)     Discuss the continuity of the function $f(x) = \begin{cases} 2x - 1 & \text{if } x < 2 \\ \dfrac{3x}{2} & \text{if } x \geq 2 \end{cases}$.

                                                                            **(6+6+6)**

**4.**

a) Find the area bounded by the curve $y = x^3$, the *y*-axis and the lines $y = 1$ and $y = 8$.

b) Evaluate $\displaystyle\int_0^{\pi/2} \frac{\sqrt{\sin x}}{\sqrt{\sin x} + \sqrt{\cos x}} dx$.

c) Find all asymptotes of the curve $4y^2 - 9x^2 = x^2y^2$.

**(6+6+6)**

**5.**

a) Solve the differential equation $\dfrac{xdy}{dx} + \cot y = 0$, $y\left(\sqrt{2}\right) = \dfrac{\pi}{4}$.

b) Test the convergence of the series $\displaystyle\sum \frac{x^{2n}}{2n}$.

c) Find the equation of the straight line passing through the origin and makes an angle which $60°$ with the line $x + \sqrt{3}y + 3\sqrt{3} = 0$.

**(8+6+4)**

**6.**

a) Find the latus rectum, the eccentricity, coordinates of foci and length of the axes of the ellipse $3x^2 + 4y^2 = 12$.

b) Solve the differential equation $(x + y)^2 \dfrac{dy}{dx} = 4$.

c) Differentiate the following function with respect to $x$ $(\tan x)^{\sec x}$.

**(8+6+4)**

**7.**

a) Find the cosine and sine of the angle between the two vectors $a = 3i + j + 2k$ and $b = 2i - 2j + 4k$. Also, find a unit vector perpendicular to both *a* and *b*.

b) Find the limit when $n \to \infty$ of the series
$$\frac{1}{n} + \frac{1}{n+1} + \frac{1}{n+2} + \cdots\cdots\cdots\cdots\cdots + \frac{1}{3n}.$$

c) Evaluate $\displaystyle\lim_{x \to 0}\left[\frac{1}{x} - \frac{1}{x^2}\log(1 + x)\right]$.

**(6+6+6)**