B3.2-R4 : DISCRETE STRUCTURE

NOTE :

1.	**Answer question 1 and any FOUR from questions 2 to 7.**
2.	**Parts of the same question should be answered together and in the same sequence.**

Time : 3 Hours **Total Marks : 100**

1. (a) Let A = {1, 2}, B = {a, b, c} and C = {c, d}, find $(A \times B) \cap (A \times C)$, $(B \cap C)$ and prove that $(A \times B) \cap (A \times C) = A \times (B \cap C)$.

(b) Using Principle of mathematical induction, prove the proposition P that the sum of first n positive integers is $\frac{1}{2}$ n(n + 1).

(c) Let R and S be two relations on set of positive integers I and R = {(a, 3a): a ∈ I} and S = {(a, a + 1) : a ∈ I}. Compute the following :

(i) RoS

(ii) RoR

(iii) RoRoR

(iv) RoSoR.

(d) A lot contain 12 items of which four are defective. Three items are drawn at random from the lot one after the other. Find the probability P that all the drawn three items are non defective.

(e) Find the truth value of $[p \rightarrow ((q \wedge (-r))) \vee s] \wedge [(-t) \leftrightarrow (s \wedge r)]$ where t is false and p, q, r, and s are true.

(f) Draw the Hasse diagram of divisors of 36 and show that set of all divisors of 36 form a lattice.

(g) Determine the validity of the following argument :

If 7 is less than 4, then 7 is not a prime number

7 is not less than 4

7 is a prime number.

(7 × 4)

2. (a) Solve the recurrence relation :

$t_n = 3t_{n-1} + 4t_{n-2}$ with $t_0 = 0$ and $t_1 = 5$

(b) Outcome of a survey among 1000 people are as follows :

(i) 595 are democrats, 595 wear glasses and 550 like ice cream.

(ii) 395 of them are democrats who wear glasses.

(iii) 350 of them are democrats who like ice cream.

(iv) 400 of them wear glasses and like ice cream.

(v) 250 people are democrats and wear glasses as well as like ice cream.

Based on above outcome, answer the following :

(i) How many of them are not democrats, do not wear glasses and do not like ice cream ?

(ii) How many of them are democrats who do not wear glasses and do not like ice cream ?

(8+5+5)

3. (a) Show that the set G = {0, 1, 2, 3, 4} is a finite abelian group of order 5 under addition modulo 5 as composition.

(b) Prove that the relation R in the set Z of integers, defined by a R b $\Leftrightarrow$ (a – b) is even, is an equivalence relation.

(9+9)

4. (a) The probability that A hits a target is $\frac{1}{3}$ and the probability that B hits a target is $\frac{1}{5}$. They both fire at the target. Find the probability that

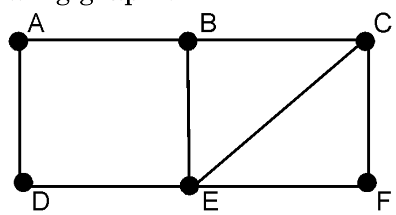
(i) a does not hit the target,

(ii) both hit the target,

(iii) one of them hit the target

(iv) neither hits the target.

(b) Consider the following graph :



(i) Find all simple paths from A to F.

(ii) Find all trails from A to F.

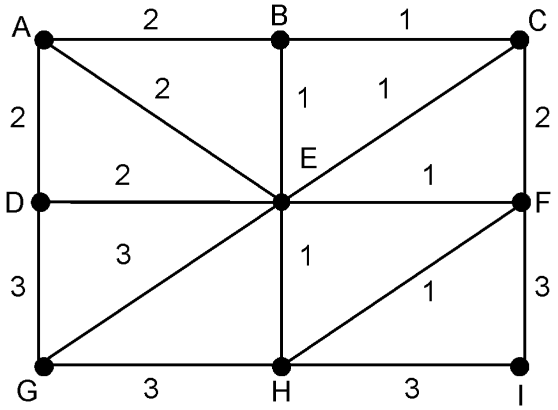
(iii) Find the distance from A to F.

(iv) Find the diameter of graph.

(v) Find all cycles which include vertex A.

([4×2]+[5×2])

5. (a) Using Prim's algorithm, find the minimum spanning tree (MST) for the following weighted graph.



(b) (i) Draw a graph with six vertices which is Eulerian but not Himiltonian.

(ii) Draw a graph with six vertices which is Hamiltonian but not Eulerian.

(10+[4+4])

6. (a) Let A = {a, b}. Describe the language L(r) where :

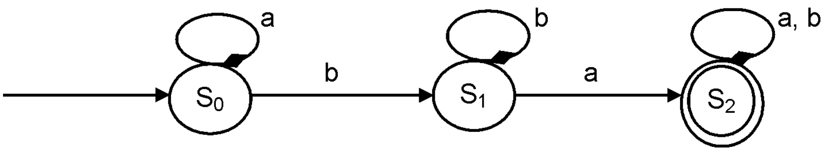
(i) r = abb*a

(ii) r = b*ab*ab*

(iii) r = ab* $\wedge$ a*

(b) Let A = {a, b}. Construct an automaton M which will accept those words from A which begin with an a followed by (zero or more) b's.

(c) Describe the words W in the language L accepted by the following automaton M :



(6+6+6)

7. (a) (i) Define Ackermann function.

(ii) Use the definition of Ackermann function to find A (1, 3).

(b) Express following Boolean expressions, E(x, y, z), as sum-of-products and then represent them in its complete sum-of-products form :

(i) $E = x(xy' + x'y + y'z)$

(ii) $E = z(x' + y) + y'$

(c) Suppose $P(n) = a_0 + a_1n + a_2n^2 + ... a_mn^m$ has degree m. Prove that $P(n) = O(n^m)$.

([2+4]+[3+3]+6)