C3-R4: MATHEMATICAL METHODS FOR COMPUTING

NOTE :

1.	**Answer question 1 and any FOUR from questions 2 to 7.**
2.	**Parts of the same question should be answered together and in the same sequence.**

Time: 3 Hours **Total Marks: 100**

1. (a) The inputto a binarycommunicationsystem, denoted by a random variable X, takes one of two values 0 or 1 with probabilities $\frac{3}{4}$and $\frac{1}{4}$respectively. Because oferrors caused by noise in the system, the output Y differs from the input occasionally. The behavior of the communication system is modeled by the conditional probabilities given below:

$P(Y=1/X=1)=3/4,\ P(Y=0/X=0)=7/8$

Find (i) $P(Y=1)$, (ii) $P(Y=0)$.

(b) A supermarket has two girls attending tosales at the counters. If the servicetimeforeachcustomer is exponential with mean 4 min and if peoplearrive in Poisson fashionat the rate of 10 per hour,

(i) What is the expectedpercentage of idletimeforeachgirl ?

(ii) If the customer has to wait in the queue, what is the expectedlength of his waiting time ?

(c) Consider the following LP model:

$Max.\ z = 5x_1 + 2x_2 + 3x_3$

s $s.t.\ x_1 + 5x_2 + 2x_3 \le b_1;\ \ x_1 - 5x_2 - 6x_3 \le b_2;\ \ x_1, x_2, x_3 \ge 0$

The following optimal tableau corresponds tospecific values of $b_1$and b_2.

Basic	x_1	x_2	x_3	x_4	x_5	**Solution**
z	0	a	7	d	e	150
x_1	1	b	2	1	0	30
x_5	0	c	–8	–1	1	10

Determine the following:

(i) The right-handside values, $b_1$and b_2.

(ii) The optimal dual solution.

(iii) The elements a, b, c, d, e.

(d) Find the Fourier Transform of $F(u)=1$ *for* $|u|<u_0$ and is 0 otherwise.

(e) At what averagerate must a clerkin a supermarket workin ordertoensure a probability of 0.90 that the customerwillnot wait longer than 12 min ? It is assumed that there is only one counter at which customers arrive in a Poisson fashion at an average rate of 15 per hourand that the length of the serviceby the clerk has an exponential distribution.

(f) Using Fourier integral, show that $e^{-ax}=\frac{2a}{\pi}\int_0^\infty \frac{cos\lambda x}{\lambda^2+a^2}d\lambda;\ a>0, x\geq 0.$

(g) If {X(t)} is a wide-sense stationaryprocess with autocorrelation $R(\tau)=Ae^{-\alpha|\tau|}$, determine the second –order moment of the random variable X(8)–X(5).

(7×4)

2. (a) If the joint pdf of (X,Y) is given by $f(x,y)=21x^2y^3, 0\leq x<y\leq 1,$ find the conditional mean and conditional variance of X given that Y=y, 0<y<1.

(b) In a singleServer queuing system with Poisson inputand exponentialservicetimes, if themeanarrivalrate is 3 calling units perhour, the expected service time is 0.25 h and the maximumpossiblenumber of calling units in the system is 2, find $P_n(n\geq 0)$, average number of calling units in the system and in the queue and average waiting time in the system and in the queue.

(9+9)

3. (a) Use the Kuhn-Tucker conditions tosolve the followingnon-linear programming problems:

$$Max.\ z=2x_1-x_1^2+x_2$$
$$s.t.\ 2x_1+3x_2\leq 6;\ 2x_1+x_2\leq 4;\ x_1,x_2\geq 0.$$

(b) A carservicingstation has 2 bays for servicing where service canbeoffered simultaneously. Because ofspace limitation, only 4 cars are acceptedfor servicing. The arrivalpattern is Poisson with 12 cars per day. The service time in both the bays is exponentiallydistributed with $\mu=8$ cars per day per bay. Find the average number of cars in the service station, the average number of cars waiting for service and the average time a car spends in the system.

(10+8)

4. (a) Using Laplace Transform, find the solution of the initialvalue problem:

$$y'' + ty' - 2y = 6 - t;\ \ y(0) = 0,\ y'(0) = 1$$

given that L[y(t)] exists.

(b) DefineRandomwalk and prove that the limitingform of random walk is the Wiener process.

(8+10)

5. (a) If X and Y are independentrandomvariablesfollowing N(8,2) andN(12, 4√3) respectively, find the value of λ such that P(2X-Y≤2λ)=P(X+2Y≥λ).

(b) Develop the BranchandBoundtreefor the followingproblemby taking $x_1$as the branching variableatnode 0.

$$Max.\ z = 2x_1 + 3x_2$$
$$s.t.\ 5x_1 + 7x_2 \le 35;\ 4x_1 + 9x_2 \le 36;\ x_1, x_2 \ge 0 \text{ and integer.}$$

(8+10)

6. (a) (i) If the number of occurrences of an event E in an interval of length t is a Poisson process {X(t)} with parameterλ and if each occurrence of E has a constantprobability p of being recorded and the recordings are independent of each other, then prove that the number N(t) of the recorded occurrences in t is also a Poisson process with parameter λp.

(ii) A radioactivesource emits particles at a rate of 5 perminutein accordance with Poisson Process. Each particle emitted has a probability 0.6 of beingrecorded. Find the probability that 10 particles are recorded in 4-min. period.

(b) A man is at an integralpointon the x-axisbetween the originand the point 3. He takes a unitstepto the right with probability 1/3 or to the left with probability 2/3, unless he is at the origin, where he takes a step to the right to reach the point 2. What is the probability that (i) he is at the point 1 after 2 walks ? (ii) he is at the point 1 in the longrun ?

(8+10)

7. (a) The inputsourceto a noisy communicationchannel is a randomvariable X over the four symbols a, b, c, d. The outputfrom this channel is a random variable Y over these same four symbols. The jointdistribution of these two random variables is as follows:

	x=a	**x=b**	**x=c**	**x=d**
y=a	1/8	1/16	1/16	1/4
y=b	1/16	1/8	1/16	0
y=c	1/32	1/32	1/16	0
y=d	1/32	1/32	1/16	0

(i) Write down the marginaldistributionfor X andcompute the marginal entropy H(X) in bits.

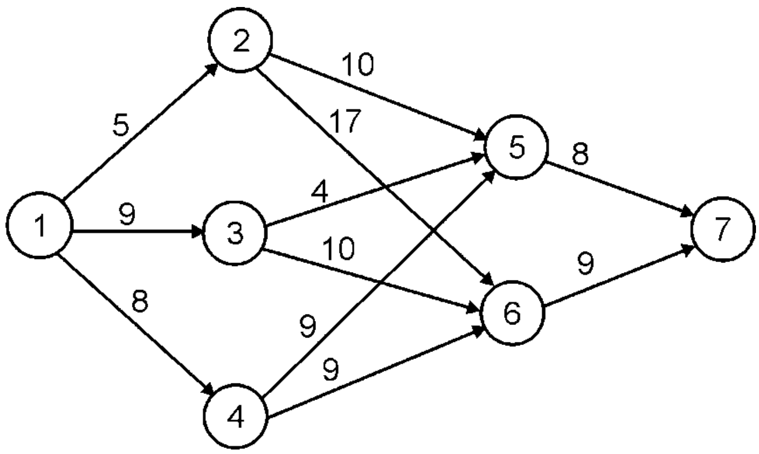
(ii) Writedown the marginaldistributionfor Y andcompute the marginal entropy H(Y) in bits.

(iii) What is the joint entropy H(X, Y) of the two randomvariables in bits?

(iv) What is the conditional entropy H(Y | X) in bits ?

(v) What is the mutualinformationI(X; Y) between the two randomvariables in bits ?

(b) For a given network, develop the backward recursive equation, anduseittofind the optimum shortest route.



(9+9)