No. of Printed Pages : **4**

Sl. No.

C3-R4 : MATHEMATICAL METHODS FOR COMPUTING

NOTE :

1. Answer question 1 and any FOUR from questions 2 to 7.

2. Parts of the same question should be answered together and in the same sequence.

Time: 3 Hours | Total Marks: 100

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1. (a) Among the digits 1, 2, 3, 4, 5 at first one is chosen and then a second selection is made among the remaining four digits. Assuming that all twenty possible outcomes have equal probabilities, find the probability that an odd digit will be selected

(i) The first time

(ii) The second time.

(b) If A and B are any two events (subset of sample space S) and are not disjoint, then prove that

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$.

(c) Find the solution by simplex method of the linear programming problem:

$$\text{Max} f(x_1, x_2) = 2x_1 + 6x_2$$

subject to

$$x_1 + x_2 \leqslant 1$$
$$2x_1 + x_2 \leqslant 2$$

$x_1 \geqslant 0,\ x_2 \geqslant 0.$

(d) Optimize the problem : $f(x_1, x_2) = x_1^3 - 3x_1x_2^2 + x_2^4$

(e) A self-service store employs one cashier at its counter. An average of 9 customers arrive every 5 minutes while the cashier can serve 10 customers in 5 minutes. Assuming Poisson distribution for arrival rate and exponential distribution for service rate, find

(i) Average number of customers in queue or average queue length.

(ii) Average time a customer waits before being served.

(f) The transition probability matrix of a three-state Markov Chain is

$$A = \begin{bmatrix} 0.3 & 0.7 & 0 \\ 0.5 & 0.4 & 0.1 \\ 0 & 0.8 & 0.2 \end{bmatrix}$$

Obtain three-step transition probability matrix.

(g) Find the inverse Laplace Transform of $f(s) = \dfrac{1}{\left(s^2 + a^2\right)^2}$ **(7x4)**

2. (a) If A and B are two independent events, then find the value of

(i) $P(A \cap \overline{B})$

(ii) $P(\overline{A} \cap B)$

Also show that

(iii) $\overline{A}$ and $\overline{B}$ are also independent events.

(b) The density function of X is given by

$$f(x) = \begin{cases} a+bx^2 & 0 \le x \le 1 \\ 0 & \text{Otherwise} \end{cases}$$

If $E(X)=\frac{3}{5}$, find a, b.

(c) A factory produces a certain type of outcomes by three types of machine. The respective daily production figures are :

Machine I : 3,000 units; Machine II : 2,500 units; Machine III : 4,500 units

Past experience shows that 1 per cent of the output produced by Machine I is defective. The corresponding fraction of defectives for the other two machines are 1.2 per cent and 2 percent respectively. An item is drawn at random from the day's production run and is found to be defective. What is the probability that it comes from the output of

(i) Machine I

(ii) Machine II

(iii) Machine III. **(6+5+7)**

3. (a) A transmitter has an alphabet consisting of five letters $\{x_1, x_2, x_3, x_4, x_5\}$ and the receiver has an alphabet consisting of four letters $\{y_1, y_2, y_3, y_4\}$. The joint probabilities for the communication are given below:

	y_1	y_2	y_3	y_4
x_1	0.25	0.0	0.0	0.0
x_2	0.1	0.3	0.0	0.0
x_3	0.0	0.05	0.1	0.0
x_4	0.0	0.0	0.05	0.1
x_5	0.0	0.0	0.05	0.0

Determine the Marginal, conditional and Joint entropies for this channel (assume0 log $0 \cong 0$).

(b) $H(X,Y) \le H(X) + H(Y)$ with equality, if and only if, X and Y are independent.

(c) Using Fourier sine integral formula, show that

$$e^{-ax} - e^{-bx} = \frac{2(b^2-a^2)}{\pi} \int_0^{\infty} \frac{u\sin(ux)}{(u^2+a^2)(u^2+b^2)}\, du,\ a \ge 0, b \ge 0$$ **(9+5+4)**

4. (a) In birth and death model (M/M/1), Let $P_n(t)=(\lambda/\mu)^n P_0$ for $n \geq 1$, denote the probability that there will be n units in the system, then show that

(i) $P_n(t)=\left(\frac{\lambda}{\mu}\right)^n\left(1-\frac{\lambda}{\mu}\right)$

(ii) Show that expected number of units in the system $L_s=\frac{\rho}{1-\rho}$

(iii) Write expected waiting line in the queue and expected waiting line in the system.

(b) A supermarket has two girls looking sales at the counters. If the service time for each customer is exponential with mean 4 minutes and if people arrive in a Poisson fashion at the rate of 10 per hour,

(i) What is the probability of having to wait for service?

(ii) If a customer has to wait, what is the expected length of the waiting time?

(c) In a railway marshalling yard, goods trains arrive at a rate of 20 trains per day. Assuming that the inter -arrival time follows an exponential distribution and the service time (the time taken to hump a train) distribution is also exponential with an average 30 minutes. Calculate the following:

(i) The average number of trains in the queue.

(ii) The probability that the queue size exceeds 10. **(6+6+6)**

5. (a) The transition probability matrix of a six-state Markov Chain is given by

$$\begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \end{array}\begin{array}{c} \begin{array}{cccccc} 1 & 2 & 3 & 4 & 5 & 6 \end{array} \\ \begin{bmatrix} 0 & \frac{1}{2} & \frac{1}{2} & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{2} & \frac{1}{2} \\ \frac{1}{4} & \frac{1}{4} & 0 & 0 & 0 & \frac{1}{2} \\ \frac{1}{4} & 0 & \frac{1}{4} & 0 & \frac{1}{2} & 0 \end{bmatrix} \end{array}$$

Graphically represent the transition probability matrix and show which is transient state and recurrent state. Also explain it is ergodic chain or not.

(b) Two manufactures A and B are competing with each other in a restricted market. Over the years, A's customers have exhibited a high degree of loyalty as measured by the fact that customers using A's product 80% of time. Also former customers purchasing the product from B have switch back to A's 60% of time.

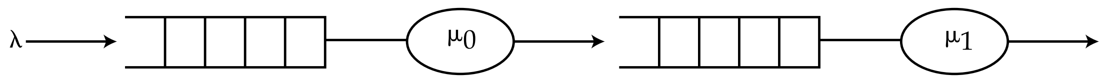
Construct and interpret the state transition matrix in terms of

(i) retention and loss

(ii) retention and gain

Also, calculate the probability of a customer purchasing A's product at the end of the second period.

(c) A repair facility shared by a large number of machines has two sequential stations with respective rates one per hour and two per hour. The cumulative failure rate of all the machines is 0.5 per hour. Assuming that the system behavior may be approximated by the two-stage tandem queue as shown in figure, determine the average repair time



(6+6+6)

6. (a) Consider the following optimizing problem:

Maximize

$$f(x_1, x_2)=4x_1+7x_2-x_1^2-2x_2^2$$

subject to constraints

$$2x_1+5x_2\leq 6$$
$$2x_1-15x_2\leq 12$$

By method of Kuhn- Tucker conditions. Explain all cases.

(b) Optimize the problem:

$$f(x_1, x_2, x_3)=2x_1^2+x_2^2+3x_3^2+10x_1+8x_2+6x_3-10$$

subject to constraints

$$x_1+x_2+x_3=20.$$

(9+9)

7. (a) Solve the differential equation $y''+2t\, y'-4y=1,\ y(0)=y'(0)=0.$

(b) Solve the integral differential equation $y'(t)-y(t)=60\int_0^t y(t)dt,\ y(0)=2$.

(9+9)