C4-R4 : ADVANCED ALGORITHMS

NOTE :
1.   Answer question 1 and any FOUR from questions 2 to 7.
2.   Parts of the same question should be answered together and in the same sequence.

Time: 3 Hours   Total Marks: 100
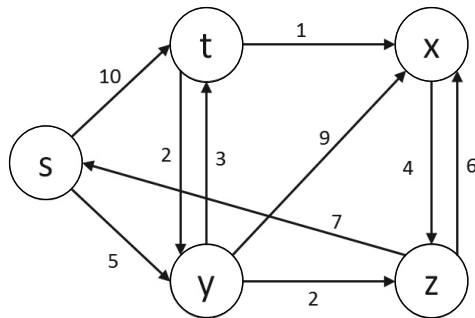
1.   (a)   Is $2^{n+1} = O(2^n)$? Is $2^{2n} = O(2^n)$?

   (b)   Write algorithm for bubble sort.

   (c)   Show that if a node in a binary search tree has two children, then its successor has no left child and its predecessor has no right child.

   (d)   Compare Kruskal's and Prim's algorithms for finding minimum Spanning Tree.

   (e)   Give a recursive algorithm MATRIX-CHAIN-MULTIPLY (A, s, i, j) that actually performs the optimal matrix-chain multiplication, given the sequence of matrices $\{A1, A2, . . . , An\}$, the s table computed by MATRIX-CHAIN-ORDER, and the indices i and j. (The initial call would be MATRIX-CHAIN-MULTIPLY (A, s, 1, n).)

   (f)   Write algorithm of bucket sort.

   (g)   What are NP, P and NP-complete problems?   **(7x4)**

2.   (a)   Find the optimum order to perform matrix multiplication $A_1A_2A_3A_4A_5A_6$ using dynamic programming method. Dimensions of matrices are given below.

   matrix   dimension
   A1 30×35
   A2 35×15
   A3 15×5
   A4 5×10
   A5 10×20
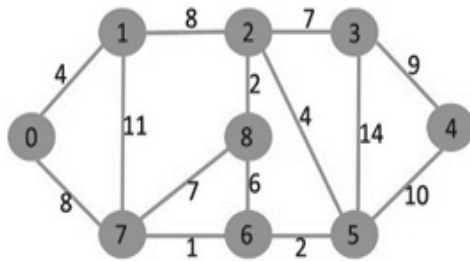   A6 20×25

   (b)   Construct max heap for input sequence 32, 33, 42, 10, 14, 19, 27, 44, 26, 31.

   (c)   Find worst case complexity of merge sort using recurrence tree method.   **(8+5+5)**

3.   (a)   Write naïve string matching algorithm. Show the comparisons the naive string matcher makes for the pattern P = 0001 in the text T = 000010001010001.

   (b)   Write Dijkstra's algorithm. Output the sequence of vertices identified by the Dijkstra's algorithm for single source shortest path when the algorithm is started at node s for the given weighted directed graph.

   (c)   Sort the given sequence of numbers in ascending order using radix sort.
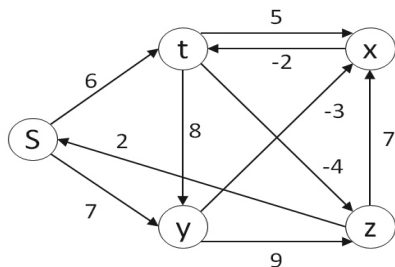   Sequence : 329, 457, 657, 839, 436, 720, 355   **(6+6+6)**

4. **(a)** Find minimum spanning tree for given graph using Kruskal's algorithm.

(b) Construct Huffman tree for given characters and also find Huffman code for them.

Character	a	b	c	d	e	f
Frequency	5	9	12	13	16	45

(c) Find length shortest path to reach to all nodes from node s.

(8+5+5)

5. **(a)** In 0-1 Knapsack problem, there are five objects, whose weights are 1, 2, 5, 6 and 7 and whose values are 1, 6, 18, 22 and 28 respectively. If one can carry a maximum of 11 units of weight, then find out the optimum value of 0-1 Knapsack using dynamic programming method.

(b) Write the algorithm of quick sort. Find worst case complexity of it using iterative method. **(8+10)**

6. **(a)** A sequence of n operations is performed on a data structure. The ith operation costs i if i is an exact power of 2, and 1 otherwise. Use aggregate analysis to determine the amortized cost per operation.

(b) Infinite coins of denominations 1, 5, 6, 8 are given. What is the minimum number of coins required to achieve a sum of 11? Use dynamic programming approach.

(c) Solve the given recurrence equation using substitution method.
$T(n) = 2T(n/2) + n$ **(8+6+4)**

7. **(a)** Apply merge sort algorithm to sort given input sequence.
Input : 14, 33, 27, 10, 35, 19, 42, 44

(b) Find the number of character comparisons done by Boyer-Moore algorithm to check the presence of string GCAGAGAG in string GCATCGCAGAGAGTATAGAGTACG.

(c) Discuss the problem of Vertex Cover and give an approximation algorithm to find Minimum Vertex Cover. **(6+6+6)**

- o 0 o -