Sl. No.

B3.2 - R4 : DISCRETE STRUCTURE

NOTE :
1. **Answer question 1 and any FOUR questions from 2 to 7.**
2. **Parts of the same question should be answered together and in the same sequence.**

Time : 3 Hours **Total Marks : 100**

1. (a) How many different elements does A^n have when A has m elements and n is a positive integer ?

(b) Let $A_i = \{1, 2, 3, \ldots, i\}$ for $i = 1, 2, 3, \ldots$.

Find : (i) $\bigcup_{i=1}^{n} A_i$. (ii) $\bigcap_{i=1}^{n} A_i$.

(c) Let S = {a, b, c, d, e, f, g}. Determine which of the following are partitions of S :

(i) P1 = [{a, c, e}, {b}, {d, g}] (ii) P2 = [{a, e, g}, {c, d}, {b, e, f}]

(iii) P3 = [{a, b, e, g}, {c}, {d, f}] (iv) P4 = [{a, b, c, d, e, f, g}].

(d) Write the dual of Boolean expression :

(i) (a * 1) * (0 + a') = 0; (ii) a + a'b = a + b.

(e) Use K map to find a minimal sum for :

$E_2 = y't' + y'z't + x'y'zt + yzt'$

(f) Suppose u = a^2b and v = b^3ab. Find :

(i) uvu; (ii) λu, uλ, uλv.

(g) A graph represented by the table :

G = [X → Y, Z, W; Y → X, Y, W; Z → Z, W; W → Z]. W, X, Y, Z are vertices of a graph.

(i) Find the number of vertices and edges in G.

(ii) Draw the graph of G.

(iii) Are there any sources or sinks ? **(7x4)**

2. (a) Given $A = \{1, 2, 3, 4\}$ and $B = \{x, y, z\}$. Let R be the following relation from A to B : $R = \{(1, y), (1, z), (3, y), (4, x), (4, z)\}$

(i) Determine the matrix of the relation.

(ii) Draw the arrow diagram of R.

(iii) Find the inverse relation of R.

(iv) Determine the domain and range of R.

(b) Let a and b be positive integers, and suppose Q is defined recursively as follows :

$Q(a, b) = 0$ if $a < b$

$= Q(a - b, b) + 1$ if $b \leq a$

(i) Find : (a) $Q(2, 5)$; (b) $Q(12, 5)$.

(ii) What does this function Q do ? Find $Q(5861, 7)$. **(9+9)**

3. (a) Use mathematical induction to show that $n! \geq 2^{n-1}$ for $n = 1, 2....$

(b) Find the truth set of each of the following predicates where the domain is the set of integers.

(i) $P(x) : x^2 < 3$ (ii) $Q(x) : x^2 > x$ (iii) $R(x) : 2x + 1 = 0$

(c) Find gcd (1064, 856), using Euclidean algorithm. **(6+6+6)**

4. (a) Let p and q be the propositions, p : You drive over 65 miles per hour. q : You get a speeding ticket. Write these propositions using p and q and logical connectives (including negations).

(i) You do not drive over 65 miles per hour.

(ii) You drive over 65 miles per hour, but you do not get a speeding ticket.

(iii) You will get a speeding ticket if you drive over 65 miles per hour.

(iv) If you do not drive over 65 miles per hour, then you will not get a speeding ticket.

(v) Driving over 65 miles per hour is sufficient for getting a speeding ticket.

(vi) You get a speeding ticket, but you do not drive over 65 miles per hour.

(vii) Whenever you get a speeding ticket, you are driving over 65 miles per hour.

(b) For each of these arguments, explain rules of inference used for each step.

(i) "Doug, a student in this class, knows how to write programs in JAVA. Everyone who knows how to write programs in JAVA can get a high-paying job. Therefore, someone in this class can get a high-paying job."

(ii) "Somebody in this class enjoys whale watching. Every person who enjoys whale watching cares about ocean pollution. Therefore, there is a person in this class who cares about ocean pollution."

(iii) "Each of the 93 students in this class owns a personal computer. Everyone who owns a personal computer can use a word processing program. Therefore, Zeke, a student in this class, can use a word processing program."

(iv) "Everyone in New Jersey lives within 50 miles of the ocean. Someone in New Jersey has never seen the ocean. Therefore, someone who lives within 50 miles of the ocean has never seen the ocean." **(9+9)**

5. (a) A factory makes custom sports cars at an increasing rate. In the first month only one car is made, in the second month two cars are made, and so on, with n cars made in the n^{th} month.

(i) Set up a recurrence relation for the number of cars produced in the first n months by this factory.

(ii) How many cars are produced in the first year ?

(iii) Find an explicit formula for the number of cars produced in the first n months by this factory.

(b) Let us suppose that G is a graph with seven vertices, such that two vertices have the degree one, three vertices have the degree two and two vertices have the degree three. Can this graph be a tree ? If your answer is yes, draw a tree with these properties. If your answer is no, prove that it couldn't be a tree.

(c) Let A be the set of all integers greater than or equal to 2, and relation R be defined by : $\forall m. n \in A: (m, n) \in R \text{ iff } m|n$

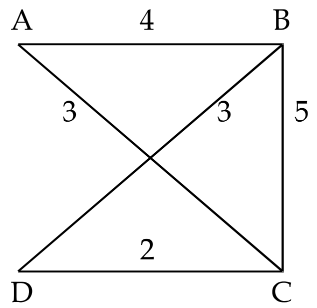
(i) 7 is a minimal element of A.

(ii) 6 is not a minimal element of A.

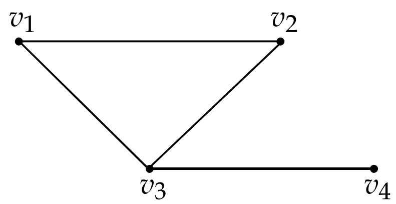
(iii) A does not have any maximal elements.

(iv) 2 is not a least element of A. **(6+6+6)**

6. (a) Find the minimal spanning tree for the following graph using Prim's algorithm :



(b) Consider the following graph :

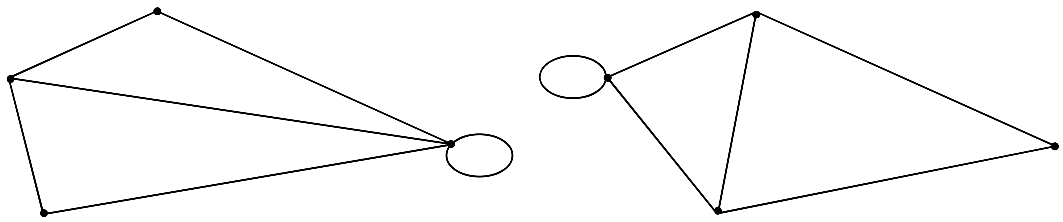


(i) Find the following, if there exist :

(A) An Eulerian path. (B) A Hamiltonian path.

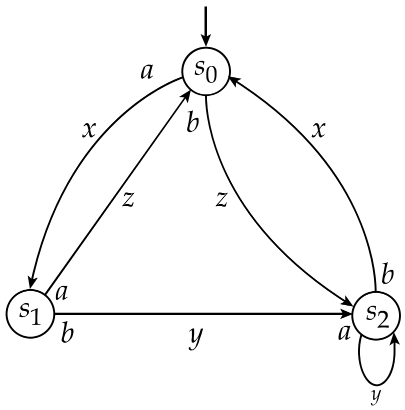
(ii) Give the adjacency matrix for the graph.

(c) Explain why the graphs below are not isomorphic.



G_1 G_2 **(6+6+6)**

7. (a) Let M be the finite state machine with input set A = {a, b}, output set Z = {x, y, z}, and state diagram D = D(M) in Fig. below :



(i) Construct the state table of M.

(ii) Find the output word v if the input is the word :

(A) $w = a^2\ b^2abab$;

(B) $w = abab^3a^2$.

(b) Consider the set Q of rational numbers, and let * be the operation on Q defined by a * b = a + b − ab

(i) Find : (a) 3 * 4; (b) 2 * (−5); (c) 7 * (1/2).

(ii) Is (Q, *) a semigroup ? Is it commutative ?

(iii) Find the identity element for *.

(iv) Do any of the elements in Q have an inverse ? What is it ? **(9+9)**

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