No. of Printed Pages : **4**

Sl. No.

C3-R4 : MATHEMATICAL METHODS FOR COMPUTING

NOTE :

1. **Answer question 1 and any FOUR from questions 2 to 7.**
2. **Parts of the same question must be answered together and in the same sequence.**

Time : 3 Hours **Total Marks : 100**

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1. (a) From 25 tickets, marked with first 25 numerals, one is drawn at random. Find the chance that

(i) it is multiple of 5 or 7 and

(ii) it is multiple of 3 or 7

(b) Give the dual of LP problem :

Minimize $z = x_1 - 2x_2 - 3x_3$,

subject to,

$$-2x_1 + x_2 + 3x_3 = 1$$
$$2x_1 + 3x_2 + 4x_3 = 4$$
$$x_1, x_2, x_3 \geq 0.$$

(c) Verify the rule of the additivity of entropies for events A, B and C with Probabilities 1/5, 4/15 and 8/15 respectively.

(d) Find the Laplace transform of $t^2 e^{-3t} \sin 4t$.

(e) Using the KKT conditions for the following non-linear programming problem verify that $x^k = \left(\frac{1}{2}, \frac{3}{2}\right)^T$ is its optimal solution

minimize $f(x) = (x_1 - 1)^2 + x_2 - 2$

subject to $h(x) = x_2 - x_1 - 1 = 0$,

$g(x) = x_1 + x_2 - 2 \leq 0$.

(f) Customers arrive at a one-window drive according to Poisson distribution with mean 10 per minute and service time per customer is exponential with mean 6 minutes. The space in front of the window including that for the serviced care can accommodate a maximum of three cars. Others can wait outside this space.

(i) What is the probability that an arriving customer can drive directly to the space in front of the window ?

(ii) How long is the arriving customer expected to wait before starting service ?

(g) Suppose that whether or not it rains today depends on previous weather conditions through the last two days. Specifically, suppose that if it has rained for the past two days, then it will rain tomorrow with probability 0.7; if it rained today but not yesterday, then it will rain tomorrow with probability 0.5; if it rained yesterday but not today, then it will rain tomorrow with probability 0.4; if it has not rained in the past two days, then it will rain tomorrow with probability 0.2, then find transition probability matrix a four-state Markov Chain. **(7x4)**

2. (a) The life time in hours of electronic tubes is a random variable having a probability density function given by :

$f(x) = a^2 x e^{-ax}, x \geqslant 0$

Compute the expected life time of such a tube.

(b) A random variable X is distributed at random between the values 0 and 1 so that its probability density function is $f(x) = kx^2(1 - x^2)$, where k is a constant. Find the value of k. Using this value of k, find its mean and variance.

(c) Obtain a Fourier series expression for :

$f(x) = x^3 \quad \text{for} \quad -\pi < x < \pi.$ **(5+7+6)**

3. (a) In a certain community, 25% of all girls are blondes and 75% of all blondes have blue eyes. Also 50% of all girls in the community have blue eyes. If you know that a girl has blue eyes, how much additional information do you being informed that she is blonde ?

(b) Calculate the steady state probabilities form the transition probability matrix given in table when the system can either start in state 1 or state 2 :

To From	State 1	State 2
State 1	0.8	0.2
State 2	0.4	0.6

(c) The joint probability density function of X and Y is given by

$$f(x, y) = \frac{6}{7}\left(x^2 + \frac{xy}{2}\right), \ 0 < x < 1, 0 < y < 2$$

(i) Verify that this is indeed a joint density function.

(ii) Compute the density function of X.

(iii) Find P{X > Y}. **(4+8+6)**

4. (a) Minimize the linear programing problem by dual simplex method as given below :

Minimize $z = 4x_1 + 2x_2 + 3x_3$

subject to constraints

$$2x_1 + 4x_3 \geqslant 5$$
$$2x_1 + 3x_2 + x_3 \geqslant 4$$
$$x_1, x_2, x_3 \geqslant 0.$$

(b) Use branch and bound technique to solve the following integer programming problem : Maximize $z = 7x_1 + 9x_2$ subject to constraints

$$-x_1 + 3x_2 \leq 6$$
$$7x_1 + x_2 \leq 35$$
$$x_1 \geqslant 0, x_2 \leq 7$$
x_1, x_2 are integers.

(c) Use two phase method to solve the following Linear programming problem :

Max $z = 2x_1 + x_2 + x_3$

Subject to
$$4x_1 + 6x_2 + 3x_3 \leq 8$$
$$3x_1 - 6x_2 - 4x_3 \leq 1$$
$$2x_1 + 3x_2 - 5x_3 \geq 4$$
$$x_1, x_2, x_3 \geq 0.$$

(5+8+5)

5. (a) Given an average arrival rate of 20 per hour, is it better for a customer to get a service at a single channel with mean service rate of 22 customers or at one of the two channels in parallel, with the mean service rate of 11 customers for each of the two channels ? Assume that both queues are (M/M/s : ∞/FIFO).

(b) A telephone exchange has two long distance operators. The telephone company finds that during the peak load, long distance calls arrive in a Poisson fashion at an average rate of 15 per hour. The length of service on these calls is approximately exponentially distributed with a mean length of 5 minutes.

(i) What is the probability that a subscriber will have to wait for his long distance call during the peak hours of the day ?

(ii) If the subscribers will wait and are serviced in turn, what is the expected waiting time ?

(c) A barber shop has space to accommodate 10 customers. The barber can service only one person at a time. If a customer comes to his shop and finds it full, he goes to the next shop. Customers randomly arrive at an average rate of 10 per hour and the barber's service time is negative exponential with an average of 5 minutes. Find P_0 and P_n.

(6+6+6)

6. (a) Find the Inverse Laplace Transform of :

$$f(s) = \frac{(s^2+16)}{(s^2+1)(s^2+4)}.$$

(b) Find the Fourier sine transform of :

$$f(x) = \frac{x}{(x^2+a^2)}.$$

(c) By using Laplace Transform, evaluate the integral :

$$\int_0^\infty e^{-t}\left(\frac{1-\cos t}{t}\right)dt.$$

(6+6+6)

7. (a) Consider a gambler who at each play of the game has probability p of winning one unit and probability $q = 1 - p$ of losing one unit. Assuming that successive plays of the game are independent, what is the probability that, starting with i units, the gambler's fortune will reach N before reaching 0 with given that $P_{00} = P_{NN} = 1$ and $P_{i,\, i+1} = p = 1 - P_{i,\, i-1}$, $i = 1, 2, \ldots, N-1$.

(b) The Markov chain of successive states has the following transition probability matrix :

$$\begin{Vmatrix} 0.6065 & 0.3033 & 0.0758 & 0.0144 \\ 0.6065 & 0.0000 & 0.3033 & 0.0902 \\ 0.0000 & 0.6065 & 0.0000 & 0.3935 \\ 0.0000 & 0.0000 & 0.6065 & 0.3935 \end{Vmatrix}$$

Calculate the limiting probabilities of four state Markov chain.

(c) Assume that the life time X and the brightness Y of an LCD bulb are being modeled as continuous random variables. Let the joint probability density function be given by :

$$f(x, y) = \lambda_1 \lambda_2 e^{-(\lambda_1 x + \lambda_2 y)},\ 0 < x < \infty,\ 0 < y < \infty.$$

Find the marginal density of X, marginal density of Y and joint distribution function of (x, y). **(8+6+4)**

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