No. of Printed Pages : **2**

Sl. No.

B0-R4 : BASIC MATHEMATICS

NOTE :

1. **Answer question 1 and any FOUR from questions 2 to 7.**
2. **Parts of the same question should be answered together and in the same sequence.**

Time : 3 Hours **Total Marks : 100**

1. (a) Find the modulus and amplitude of $\frac{(1+i)\left(1+\sqrt{3i}\right)}{1-i}$.

(b) Evaluate $\lim_{x\to 0}\frac{\tan x-\sin x}{x^3}$.

(c) Show that the matrix $A=\begin{bmatrix} 3 & 7+4i & -2+5i \\ 7-4i & -2 & 3+i \\ -2-5i & 3-i & 4 \end{bmatrix}$ is a Hermitian matrix.

(d) Test the convergence of the series $\frac{1}{\sqrt{1.2}}+\frac{1}{\sqrt{2.3}}+\frac{1}{\sqrt{3.4}}+...$

(e) Prove that $\int_0^{\frac{\pi}{4}}\log(1+\tan\theta)\,d\theta=\frac{\pi}{8}\log 2$.

(f) Solve $\left(1+x^2\right)\frac{dy}{dx}+y=\tan^{-1}x$.

(g) Find the point on the curve $x=a(\theta+\sin\theta)$, $y=a(1-\cos\theta)$, where the tangent is inclined at an angle $\frac{\pi}{4}$ to the x-axis. **(7x4)**

2. (a) Solve the following system of simultaneous linear equations by Gauss elimination method :

$4x+y+z=4$
$x+4y-2z=4$
$3x+2y-4z=6$

(b) If $P=\begin{bmatrix} 1 & 1 \\ -2 & 1 \end{bmatrix}$ and $A=P\begin{bmatrix} 2 & 0 \\ 0 & 5 \end{bmatrix}P^{-1}$, find A.

(c) If $\frac{x}{x-y}=\log\frac{a}{x-y}$, prove that $\frac{dy}{dx}=2-\frac{x}{y}$. **(6+6+6)**

3. (a) Obtain the eigen values and the eigen vectors for the matrix $A = \begin{bmatrix} 2 & 0 & 1 \\ 1 & 2 & 0 \\ 1 & 0 & 2 \end{bmatrix}$

(b) Show that the length of the arc of the parabola $y^2 = 4ax$ intercepted between vertex and an extremity of the latus rectum is a $\left[\sqrt{2} + \log\left(1 + \sqrt{2}\right)\right]$. **(9+9)**

4. (a) If $x = \cos\theta + i \sin\theta$ and $y = \cos\varphi + i \sin\varphi$. Show that the value of

$x^m y^n + \frac{1}{x^m y^n}$ is $2\cos(m\theta + n\varphi)$.

(b) Find the asymptote of the following curve $x^3 + 2x^2y - xy^2 - 2y^3 + xy - y^2 - 1 = 0$.

(c) Evaluate $\int \frac{dx}{(x-1)^2(x-2)}$. **(6+6+6)**

5. (a) Find the maximum and minimum values of the function $f(x) = 4x^{-1} - (x-1)^{-1}$ for all $x \in R - \{0, 1\}$.

(b) Examine the continuity of the function

$$f(x) = \begin{cases} (1 + 2x)^{\frac{1}{x}}, & \text{when } x \neq 0 \\ & \text{at } x = 0 \\ e^2, & \text{when } x = 0 \end{cases}$$

(c) Expand the polynomial $f(x) = x^3 - 2x^2 + 3x + 5$ in the positive integral powers of $(x-2)$. **(6+6+6)**

6. (a) Find the area of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$.

(b) Find the equation of ellipse under the conditions :
Center at (0, 0); Latus rectum = 10 and distance between foci = Length of minor axis. **(9+9)**

7. (a) Solve $x^2 \frac{d^2y}{dx^2} + x\frac{dy}{dx} - y = x^3$.

(b) If $\hat{a}$ and $\hat{b}$ are unit vectors and θ is the angle between them, Show that
$\sin \theta/2 = 1/2 \left|\hat{a} - \hat{b}\right|$.

(c) Find the parametric equation of the line through the point $(1, -2, 2)$ and perpendicular to the plane $x + 2y - 3z = 5$. **(6+6+6)**

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