**B0-R4: BASIC MATHEMATICS**

**NOTE:**

1. **Answer question 1 and any FOUR from questions 2 to 7.**
2. **Parts of the same question should be answered together and in the same sequence.**

**Time: 3 Hours** **Total Marks: 100**

**1.**

a) Express $(5 - 3i)^2$ in the form a + *i*b.

b) Find $\lim_{x \to 1} \frac{x^2 + 1}{x + 100} + \lim_{x \to 0} \frac{\sin 2x}{x}$

c) If $\underline{p} + \underline{q} + \underline{r} = \underline{o}$, |p| = 3, |q| = 5, |r| = 7, find the angle between $\underline{p}$ and $\underline{r}$.

d) Draw the graph of y = x + |x| and compute $\int_{-1}^{1} \{x + |x|\} dx$.

e) Evaluate the area between the parabola $y = x^2$ and the lines x = 1 and x = -1.

f) If x = cosθ + θsinθ, y = sinθ – θcosθ, find $\frac{dy}{dx}$ at $\theta = \frac{\pi}{4}$.

g) Test the convergence of the series $1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \ldots$

for all x.

**(7x4)**

**2.**

a) Show that the matrix $A = \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}$

satisfies the equation $\mathbf{A}^2 - 4\mathbf{A} + \mathrm{I} = \mathbf{O}$ where I is 2x2 identity matrix and **O** is 2x2 zero matrix. Using this equation, compute $\mathbf{A}^{-1}$.

b) Evaluate the determinant

$$\Delta = \begin{vmatrix} b+c & a & a \\ b & c+a & b \\ c & c & a+b \end{vmatrix}$$

**(10+8)**

**3.**

a) Evaluate the following limit $\lim_{x \to 0} \frac{\tan 2x - \sin 2x}{x^3}$.

b) Differentiate the following function with respect to x;
$(\sin x)^{\ln x}$.

c) If sin y = x cos (a + y), show that $\frac{dy}{dx} = \frac{\cos^2(a + y)}{\cos a}$, hence compute $\frac{dy}{dx}$ when x = 0.

**(6+6+6)**

**4.**

a) State Rolle's theorem. Discuss the applicability of Rolle's theorem for the function on the indicated interval f(x) = |x| on [-1, 1].

b) Find the slope of the tangent to curve $x^2 + 3y + y^2 = 5$ at (1, 1).

c) Find the maximum profit that a company can make if the profit function is given by
$p(x) = 41 + 24x - 18x^2$.

**(6+6+6)**

**5.**

a) Evaluate

i) $\int \frac{e^{3x}}{1+e^{3x}} dx$

ii) $\int \frac{1}{x^2+4x+8} dx$

b) Find the area of the region bounded by the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$.

c) Form the differential equation of the family of curves represented by $c(y + c)^2 = x^3$.

**(6+6+6)**

**6.**

a) Show that the series

$$\sum_{n=1}^{\infty} \frac{n}{2n+1}$$

converges.

b) If $\underline{u}=\hat{i}+2\hat{j}-2\hat{k}$ and $\underline{v}=3\hat{i}+\hat{k}$,
Find $\underline{u}$ X $\underline{v}$.

c) Compute $(1+\sqrt{3}\,i)^9$, $i=\sqrt{-1}$.

d) Draw the graph of $y = xe^{-x}$, $x > 0$.

**(6+4+4+4)**

**7.**

a) Find all the points of local maxima and minima of the function
$f(x) = x^3 - 6x^2 + 9x - 8$

b) Find the point on the curve $y = 2x^2 - 6x - 4$ at which the tangent is parallel to x-axis.

**(12+6)**