B3.2-R4: DISCRETE STRUCTURE

NOTE:

1.	**Answer question 1 and any FOUR from questions 2 to 7.**
2.	**Parts of the same question should be answered together and in the same sequence.**

Time: 3 Hours **Total Marks: 100**

1.

a) Let A={a, e, f, g i}, B={b, d, e, g, h}, c={d, e, f, h, i}. Find
 i) $(A \cup B) \cap C$
 ii) $A – (B – C)$

b) Find gcd {2076, 1024} using the Euclidean algorithm.

c) Find the general expression for a_n, where $a_1 = 0$ and $a_n = a_{n-1} + (n – 1)$, $n \geq 2$.

d) Let **X** = {a, b, c}. Define a relation R on **X** x **X** as R={(a, b), (b, b), (c, a), (c, c)}. Show that $R^2=R^3$.

e) Draw a graph whose adjacency matrix is

$$\begin{array}{c} \\ a \\ b \\ c \\ d \\ e \end{array} \begin{array}{c} \begin{array}{ccccc} a & b & c & d & e \end{array} \\ \begin{bmatrix} 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 1 & 1 \\ 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 & 1 \end{bmatrix} \end{array}$$

f) Let $\Sigma = \{0, 1\}$, A={0, 01}, B={λ, 1, 110}. Find the concatenations AB and BA.

g) Evaluate the Boolean expression (x + y + z) (x + y’ + z) at the ordered triplet (1, 0, 1), (0, 0, 1), (1, 1, 0) and (1, 1, 1).

(7x4)

2.

a) Using the Karnaugh map, simplify the Boolean expression
 wxyz + wxyz’ + wxy’z’ + wxy’z + w’xyz + w’xy’z

b) A survey among 100 students shows that, of the three ice cream flavours vanilla, chocolate and strawberry, 50 students like vanilla, 43 like chocolate, 28 like strawberry. There are 13 students who like vanilla and chocolate, 11 like chocolate and strawberry, 12 like strawberry and vanilla, and 5 like all the three. Find the number of students who like each of the following flavours:
 i) Chocolate but not strawberry.
 ii) Vanilla or Chocolate but not strawberry.

(10+8)

3.

a) Prove that the number of odd degree vertices in a graph is an even integer.

b) If we select any group of 1000 students in a campus, show that at least 3 of them must have the same birthday.

(10+8)

4.

a) Let H be a subgroup of a group G and $K = \{g \in G \mid gHg^{-1} = H\}$. Prove that K is a subgroup of G containing H.

b) Let $f(x) = \begin{cases} x+2 & \text{if } x \leq 4 \\ x-3 & \text{if } x > 4 \end{cases}$ and $g(x) = \begin{cases} x^2 & \text{if } x \leq 5 \\ 2x-1 & \text{if } x > 5 \end{cases}$. Find (gof)(x).

(10+8)

5.

a) Let N = {A, B, σ}, T = {a, b}, and P = {σ → aA, A → bA, A → a}. Identify the language L(G) generated by the grammar G = {N, T, P, σ}.

b) Solve the recurrence relation

$$a_n = 6a_{n-1} - 9a_{n-2},\ n \geq 2,$$

where $a_0 = 2$, $a_1 = 3$.

c) Let f be a permutation defined on a set {1, 2, 3, 4, 5, 6} by $f = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 2 & 5 & 1 & 6 & 4 & 3 \end{pmatrix}$.

Find f^2.

(7+7+4)

6.

a) Prove that n^2-1 is divisible by 8 whenever n is an odd positive integer.

b) Solve the congruence relation 2x ≡ 7(mod 17).

(10+8)

7.

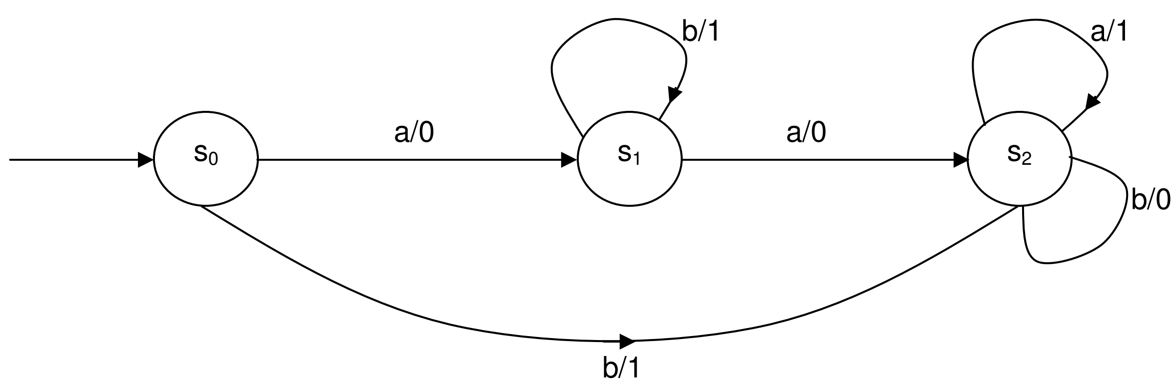
a) Determine which of the following graphs are Hamiltonian

i) $K_{2,3}$

ii) $K_{2,4}$

If the graph is not Hamiltonian, does it contain a Hamiltonian path? Give reasons.

b) Construct the transition table of the finite state machine whose transition diagram is as follows:



c) Find the coefficient of x^3y^4 in the expansion of $(x+y)^7$.

(8+6+4)