B0-R4: BASIC MATHEMATICS

NOTE:

1. **Answer question 1 and any FOUR from questions 2 to 7.**
2. **Parts of the same question should be answered together and in the same sequence.**

Time: 3 Hours **Total Marks: 100**

1.

a) Find the real and imaginary part of expression $(\sec\theta + i\tan\theta)^2 - 1$

b) Find the limit $\lim_{x\to 2}\left(\frac{x}{x-2} \quad \frac{4}{x^2-2x}\right)$.

c) Find the inverse of $A=\begin{bmatrix}0 & 1 & 1\\ 1 & 2 & 0\\ 3 & -1 & 4\end{bmatrix}$.

d) Show that the vectors

$$\vec{A}=2\hat{i}-\hat{j}+\hat{k}$$
$$\vec{B}=\hat{i}-3\hat{j}-5\hat{k}$$
$$\vec{C}=3\hat{i}-4\hat{j}-4\hat{k}$$

from the sides of a right-angled triangle.

e) Evaluate the following $\int \frac{e^{m\tan^{-1}x}}{1+x^2}\,dx$.

f) If $y = 4\cos x - 2\sin x$, prove $\frac{d^2y}{dx^2}=-y$.

g) Test the convergence of the series $\frac{x}{1^2}+\frac{x^2}{2^2}+\frac{x^3}{3^2}+\ldots$.

(7x4)

2.

a) Show that if $A=\begin{bmatrix}1 & 3\\ 2 & 6\end{bmatrix}$ and $B=\begin{bmatrix}-1 & 4\\ 2 & 1\end{bmatrix}$ then $(A+B)^2 \neq A^2 + 2AB + B^2$.

b) Solve by using Cramer's rule:

$$2x + 3y + 4z = 29$$
$$3x - y + z = 10$$
$$x + 2y + 2z = 16$$

(9+9)

3.

a) Find all the asymptotes of the curve $y^2(x - b) = x^3 + a^3$.

b) Show that if $y = \sin(m\sin^{-1}x)$, then $\left(1-x^2\right)\frac{d^2y}{dx^2}-x\frac{dy}{dx}+m^2y=0$.

c) Find two positive numbers x and y such that $x + y = 60$ and xy^3 is maximum.

(8+6+4)

4.

a) Evaluate $\int e^{2x} \sin 3x \, dx$.

b) Find the equation of ellipse with centre (5, 4), semi major axis 8 and eccentricity $\frac{1}{2}$.

c) Find equation of plane passing from (3, 4, 5), (1, 2, 3), (8, 6, 10).

(6+6+6)

5.

a) Expand the following function by Maclaurin's theorem upto four terms f(x) = cos x.

b) A man is walking at the rate of 6.5 km/hr. towards the foot of a tower 120 m high. At what rate is he approaching the top of the tower when he is 50 m away from the tower?

c) Integrate the following: $\int \frac{1}{1+3\sin^2 x + 8\cos^2 x} \, dx$.

(6+6+6)

6.

a) The equation of electromotive force for an electric circuit containing resistance and self inductance is E = Ri + L $\frac{di}{dt}$, where i is current, E is the electromotive force given to the circuit, R, the resistance and L, the coefficient of Induction. Find the current at time t when E = 0.

b) Find the equation of tangent to the curve $y^2 = x^2 + 2x$, which is parallel to straight line y + 3x + 2 = 0.

c) If $\left|\vec{a}+\vec{b}\right| = \left|\vec{a}-\vec{b}\right|$, then show that $\vec{a}$, $\vec{b}$ are perpendicular.

(8+6+4)

7.

a) Find the area of the region $\{(x, y) : x^2 \le y \le x\}$.

b) Find the equation of circle concentric with the circle $2x^2 + 2y^2 + 4x + 6y - 20 = 0$ and of radius 20.

c) Solve $\frac{dy}{dx} = \frac{y}{x}$ if y = 2 when x = 3.

(6+6+6)