B3.2-R4: DISCRETE STRUCTURE

NOTE:

1.	**Answer question 1 and any FOUR from questions 2 to 7.**
2.	**Parts of the same question should be answered together and in the same sequence.**

Time: 3 Hours **Total Marks: 100**

1.

a) Let Z^+ be the set of all positive integers. Define a function f on Z+ as
$$f(n)=\begin{cases} n/2 & \text{if } n \text{ is even} \\ (1-n)/2 & \text{if } n \text{ is odd} \end{cases}$$
show that f is a 1 – 1, onto function.

b) Prove that (p Λ q) Λ (p → q) q is a tautology.

c) Simplify the Boolean expression $[(x_1+x_2)+(\bar{x}_{1+x_2})]\ x_1\bar{x}_2$.

d) A graph has degree sequence 5,5,4,4,3,3,3,3. How many edges does it have?

e) Let A and B be nonempty sets. Prove that if A x B = B x A then A = B.

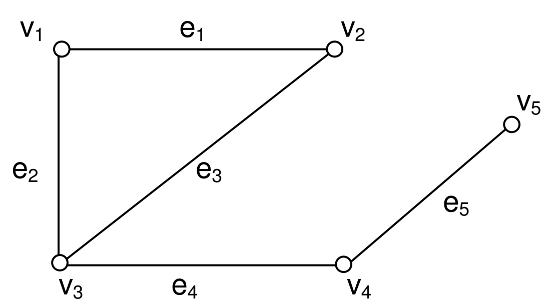
f) Find one integer x, 0 ≤ x ≤ 595, satisfying the following congruence relation:
16 x Ξ 301 (mod 595).

g) Find the deterministic finite automata for the following language:
L = {x ∈ {0, 1}* |x does not contain more than three 0's}.

(7x4)

2.

a) Consider the following graph



Write the incidence matrix and adjacency matrix of the graph.

b) Suppose a department contains 10 men and 15 women. How many ways are there to form a committee with 6 members if it must have the same number of men and women?

c) Solve the recurrence relation $a_n = 4a_{n-1} - 4a_{n-2}$, n > 2, when it is given that $a_1 = 1$ and $a_2 = 0$.

(4+6+8)

3.

a) A computer network consists of six computers. Each computer is directly connected to either none or more of the other computers. Use pigeonhole principle to show that there are at least two computers in the network that are directly connected to the same number of other computers.

b) Let G be a planar graph with more than 3 vertices and e edges. Then prove that e ≤ 3v – 6.

c) Use mathematical induction to prove that $1+\frac{1}{4}+\frac{1}{9}+...+\frac{1}{n^2}<2-\frac{1}{n},\ n>1.$

(4+6+8)

4.

a) As $\rightarrow$ Let A = {2, 3, 6, 12} and R and S be the relations on A defined as follows

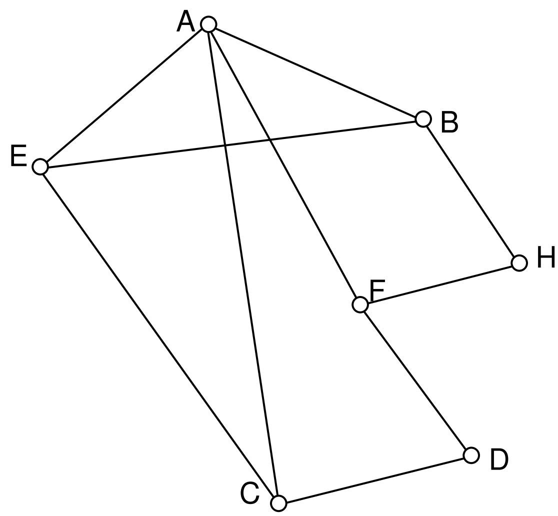
x Ry if 2 divides (x - y).
x Sy if 3 divides (x - y).

Compute

i) $R \cup S$

ii) $R \cap S$

b) Determine whether the graph given below has a Hamiltonian path? Does it has a Hamiltonian cycle? Explain.



c) Let G be a set of real numbers. Define a composition * on G as follows

$$a * b = \frac{ab}{3}, \; a, b \in G.$$

(6+6+6)

5.

a) Find integers m and n such that 512m + 320n = 64.

b) Draw the Hasse diagram for the partial order "divided" on the set S = {2, 3, 4, 6, 8, 12, 18}. Does S have an upper bound and at lower bound? Write minimal elements and maximal elements of S?

c) Show that the statement

$$[p \vee [(\neg r) \rightarrow (\neg s)] \vee [(s \rightarrow ((\neg t) \vee p) \vee ((\neg q) \rightarrow r)]$$

is neither a tautology nor a contradiction, where t means truth.

(4+6+8)

6.

a) Show that a group of order 4 is abelian.

b) Show that $n! = O(n^n)$, but $n^n \neq O(n!)$.

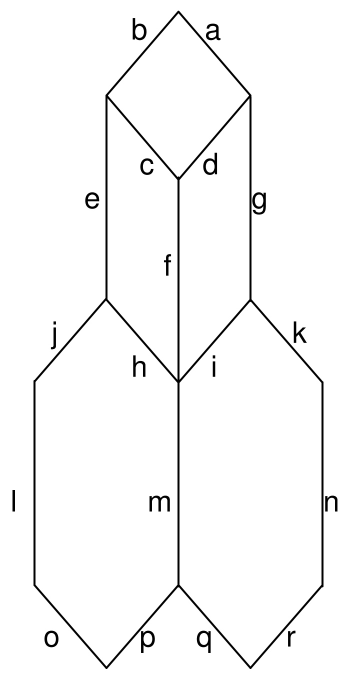
c) In a group of 97 students, the number taking English is twice the number taking Maths. 53 students take exactly one of these subjects and 15 are taking neither course. How many students are taking Maths? How many are taking English?

(4+6+8)

7.

a) Find the generating function of sequence 3, 1, 3, 1, 3, 1, …

b) Find all the cut sets in the following graph:



c) Sort the list 3, 1, 7, 2, 5, 4 into increasing order using the bubble sort. Explain the steps involved very clearly.

(4+6+8)