B4.1-R4: COMPUTER BASED STATISTICAL & NUMERICAL METHODS

NOTE:

1.	**Answer question 1 and any FOUR from questions 2 to 7.**
2.	**Parts of the same question should be answered together and in the same sequence.**

Time: 3 Hours **Total Marks: 100**

1.

a) A fair die is rolled three times and the scores added. What is the probability that sum of scores is 6?

b) It is known that all items produced by a certain machine will be defective with probability 0.1, independently of each other. Find the probability that in a sample of three items, at most one will be defective.

c) If $u = 3v^7 - 6v$, find the percentage error in u at v = 1 if the error in v is 0.05.

d) Find from the following table, the area bounded by the curve and the x-axis from x = 7.47 to x = 7.52.

x	7.47	7.48	7.49	7.50	7.51	7.52
f(x)	1.93	1.95	1.98	2.01	2.03	2.06

e) If X and Y are independent variables, find Cov(X, Y).

f) If X is a normal variate with mean μ and variance σ^2, find variance of Y = 2X + 1.

g) The two lines of regression are given as

$$X + 2Y - 5 = 0$$
$$2X + 3Y = 8$$

Compute the mean value of X and Y.

(7x4)

2.

a) Use bisection method to find the positive root of 30 correct to two decimal place.

b) Perform four iterations of the Newton-Raphson method to find the smallest positive root of the equation

$$f(x) = x^3 - 5x + 1 = 0$$

The smallest positive root lies in the interval (0, 1). Take the initial approximation as $x_0 = 0.5$.

c) Given f(2) = 4, f(2.5) = 5.5, find the linear interpolating polynomial using Lagrange interpolation.

(6+6+6)

3.

a) Evaluate $\int_0^1 \frac{dx}{1+x}$

by dividing the interval of integration into eight equal parts. Hence find $\log_e 2$ approximately.

b) Use Gauss elimination to solve

$$\begin{aligned} 2x + y + z &= 10 \\ 3x + 2y + 3z &= 18 \\ X + 4y + 9z &= 16 \end{aligned}$$

(8+10)

4.

a) Suppose that X is a continuous random variable whose probability density function (pdf) is given by $f(x)=\begin{cases} c(4x-2x^2) & 0<x<2 \\ 0 & otherwise \end{cases}$

i) What is the value of C?

ii) Find P(X > 1).

b) The joint pdf of the bivariate random variable (X, Y) is

$$f(x,y)=\begin{cases} \frac{1}{8}(x+y) & 0\le x,y\le 2 \\ 0 & otherwise \end{cases}$$

Find marginal pdf's of X and Y. Are X and Y independent?

(10+8)

5.

a) Suppose that the number of typographical errors on a single page of a book has Poisson distribution with parameter λ = ½. Calculate the probability that there is atleast one error on a page.

b) Compute Var X where X represents the outcome when we roll a fair dice.

c) X is a Binomial variate with parameters n and p. The mean and variance of X are 3 and 2.1, respectively. Find n and p.

(6+6+6)

6.

a) The yield of chemical A in litres of a fermentation process is related to the temperature during fermentation. The results of a sequence of experiments are as follows:

X (ºC)	35	40	45	50	55	60
Y (litres)	20.2	23.1	23.2	23.6	25.8	26.3

i) Find the regression of Y and X.

ii) Estimate the average yield if the fermentation temperature is 48 ºC.

b) The number of defects found on a roll of carpet has a Poisson distribution with parameter λ. If four rolls of carpet are inspected and found to have 12, 4, 9 and 15 defects, respectively, find maximum likelihood for λ.

(10+8)

7.

a) The following data come from a normal population having standard deviation 4.

105, 108, 112, 121, 100, 105, 99,
107, 112, 122, 118, 105

Use them to test the null hypothesis that the population mean is less than or equal to 100 at

i) 5 percent level of significance.

ii) 1 percent level of significance.

iii) What is the p value?

b) The manufacturer of a new fibre glass tyre claims that the average life of a set of its tyres is atleast 50,000 km. To verify this claim, a sample of 8 sets of tyres was chosen and the tyres subsequently were tested by a consumer agency. If the resulting values of the sample mean and sample variance were, respectively 47.2 and 3.1 (in 1,000 kms.), test the manufacturer's claim at 5% level of significance.

(9+9)