BE10-R4: APPLIED OPERATIONS RESEARCH

NOTE:

1. **Answer question 1 and any FOUR from questions 2 to 7.**
2. **Parts of the same question should be answered together and in the same sequence.**

Time: 3 Hours **Total Marks: 100**

1.

a) Solve the following linear programming problem by graphical method:

Maximize: $Z = 10x_1 + 15x_2$

Subject to $2x_1 + x_2 \le 26$

$x_1 - x_2 \ge -5$

$x_1,\ x_2 \ge 0$

b) A machine shop has just stocked 10 spare parts for the repair of a machine. Stock replacement of size 10 pieces occurs every 7 days. The time between breakdowns is exponential with mean 1 day. Determine the probability that the machine will remain broken for 2 days, because no spare parts are available.

c) Find initial basic feasible solution of following transportation problem using matrix minima method.

	1	2	3	4	Supply
1	4	6	8	8	40
2	6	8	6	7	60
3	5	7	6	8	50
Demand	20	30	50	50	

d) Six jobs are to be processed on two machines in order A and B. Each machine can process only one job at a time. The processing time (in hours) are given as follows:

	J1	J2	J3	J4	J5	J6
M1	10	12	13	7	14	5
M2	15	11	8	9	6	7

Find the sequence in which the jobs should be processed in order to minimize the total elapsed time.

e) Using the following data, obtain the economic order quantity (EOQ) and the total variable cost associated with the policy of ordering quantities of that size

Annual Demand: 20,000

Ordering Cost: Rs 150 per order

Inventory Carrying cost: 24% of average inventory value.

f) Write the dual of the following linear programming problem:

Maximize $Z = 5x_1 + 12x_2 + 4x_3$

Subject to $x_1 + 2x_2 + x_3 \le 10$

$2x_1 - x_2 + 3x_3 = 8$

$x_1,\ x_2,\ x_3 \ge 0$

g) In a queue if exponential arrival rate of customers is 8 per hour and Poisson service rate on a single window is 9 per hour, then find the following:

i) length of the system.

ii) waiting time in system.

(7x4)

2.

a) For the following two person zero sum game, find the optimal strategy for each player and the value of the game.

		Player B			
		B_1	B_2	B_3	B_4
Player A	A_1	0	-1	3	5
	A_2	-5	2	4	5
	A_3	-2	-3	-4	-2

b) Find optimal solution of the following assignment problem using Hungarian method:

Workers	Job			
	A	B	C	D
1	45	40	51	67
2	57	42	63	55
3	49	52	48	64
4	41	45	60	55

(9+9)

3.

a) A tailor specializes in ladies' dresses. The number of customers approaching the tailor appears to be Poisson distributed with a mean of 6 customers per hour. The tailor attends the customers on a first-come-first-served basis and the customers wait if the need be. The tailor can attend the customers at an average rate of 10 customers per hour with the service time exponentially distributed. Find

i) the probability that the queuing system is ideal.

ii) the probability associated with the number of customers (0 through 5) in the queuing system.

iii) What is the expected number of customers in the tailor shop?

iv) What is the expected number of customers waiting for tailor's service?

b) Solve the following linear programming problem by Simplex (Big-M) method:

Maximize $Z = 20x_1 + 30x_2$

Subject to $2x_1 + x_2 \le 40$

$4x_1 - x_2 \le 20,$

$x_1 \ge 30,$

$x_1, x_2 \ge 0$

(9+9)

4.

a) A firm works 40 hours a week and has a capacity of overtime work to the extent of 20 hours in a weak. It has received seven orders to be processed on three machines A, B, and C, in the order A, B, C to be delivered in a week's time from now. The process time (in hours) are recorded in the given table.

Job	:	1	2	3	4	5	6	7
Machine A	:	7	8	6	6	7	8	5
Machine B	:	2	2	1	3	3	2	4
Machine C	:	6	5	4	4	2	1	5

The manager, who, in fairness insist on performing the jobs in the sequence in which they are received, is refusing to accept an eighth order because according to him, the eight jobs would require a total of 61 hours for processing, which exceeds the firm's capacity. Advice the manager for getting optimal output sequence of seven jobs on three machines.

b) Consider the function

$f(x_1, x_2, x_3) = 2x_1^2 + x_2^2 + 2x_3^2 + x_1 x_3 + 4x_1$

Compute the stationary point defined by vector $x_0 = (x_{01}, x_{02}, x_{03})$, and show that the Hessian matrix $\nabla^2 f$ is positive definite at x_0.

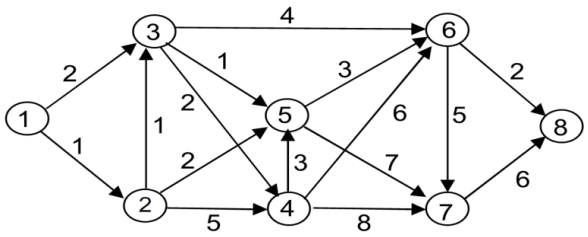
(10+8)

5.

a) Using Vogel's approximation method find initial basic feasible solution of the following transportation problem and hence optimize the solution by MODI method.

	D_1	D_2	D_3	D_4	Supply
S_1	6	4	9	1	40
S_2	20	6	11	3	40
S_3	7	1	0	14	50
S_4	7	1	12	6	80
Demand	90	30	50	50	

b) The network in the following figure gives the distance in miles between pairs of cities 1, 2, … and 8. Use Dijkstra's algorithm to find the shortest route between Cities 1 and 8:



(10+8)

6. The owner of a chain of fast-food restaurants is considering a new computer system for accounting and inventory control. A computer company sent the following information about the system installation:

Activity	Activity Description	Immediate Predecessor	Expected time (Days)		
			Most optimistic time	Most likely time	Most pessimistic time
A	Select the comp. model	--	4	6	8
B	Design input/output system	A	5	7	15
C	Design monitoring system	A	4	8	12
D	Assemble comp. hardware	B	15	20	25
E	Develop the main programs	B	10	18	26
F	Develop input/output routines	C	8	9	16
G	Create data base	E	4	8	12
H	Install the system	D, F	1	2	3
I	Test and implement	G, H	6	7	8

i) Construct a network diagram for this problem.

ii) Determine the critical path and compute the expected project completion time.

(18)

7.

a) Solve the following integer linear program by branch and bound method:

$$\text{Max } 2x + 3y$$
$$\text{subject to } \quad 8x + 5y \leq 40$$
$$x + 2y \leq 8$$
$$x, y \geq 0$$
$$x \text{ and } y \text{ are integers.}$$

b) A hardware store procures and sales hardware items. Information on an item is given below:
Expected annual sales = 8000 units
Ordering cost = Rs 180 per order
Holding cost = 10% of the average inventory value
The item can be purchased according to the following schedule:

Lot Size	Unit Price (Rs)
1-999	22.00
1000-1999	20.00
2000 and above	19.00

Using above data, determine the best order size.

(9+9)